

Optimization of Geometric Multigrid for Emerging Multi- and Manycore Processors

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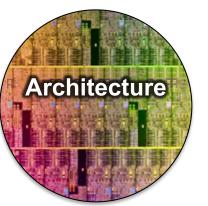


Pathways to Performance

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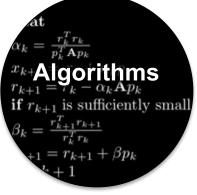
- Transistor density, frequency, energy
- DRAM technology DDR3 vs. GDDR5



- coherent caches vs. local stores
- Prefetching vs. Multithreading



- cache blocking, loop fusion, SIMDization
- programming models



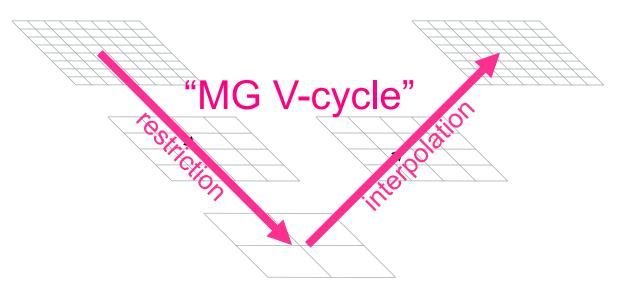
- high-order discretizations
- Linear Solvers
- comm.-avoiding

- In this talk, we...
 - Examine a variety of techniques from these areas to maximize performance of the geometric multigrid v-cycle.
 - Evaluate Intel's new Xeon PhiTM (Knights Corner) coprocessor
 - Differentiate the benefits of technology (GDDR) and architecture (coherent caches)
 - Quantify the benefits of optimization and communication-avoiding.

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- Linear Solvers (Ax=b) are ubiquitous in scientific computing...
 - Combustion, Climate, Astrophysics, Cosmology, etc...
- Multigrid exploits the nature of elliptic PDEs to provide a hierarchical approach with O(N) computational complexity.
 - Geometric Multigrid is specialization in which the linear operator (A) is simply a stencil on a structured grid (i.e. *matrix-free*)
 - Applicable to small (yet very important) range of linear systems

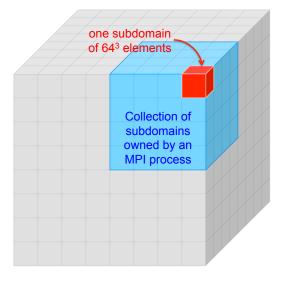




Benchmark: Specification

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- ✤ 256³ grid/node at the finest resolution...
 - fits into KNC or GPU device DRAM =
 - no PCIe transfers are required.
- subdivided into 64³ subdomains ("boxes")
 - designed to mimic some of the challenges associated with AMR MG
 - explicit communication of subdomain boundaries (ghost zones).
- Truncated V-cycle (stops at 4³ subdomains)
- Gauss Seidel, Red-Black ("GSRB") relaxation...
 - 2 GSRB's (4 stencil sweeps) at each level down the v-cycle
 - 24 GSRB's at the bottom (sufficient for single-node convergence)
 - 2 GSRB's at each level back up the v-cycle
- Fixed 10 V-cycles





helmholtz = a*alpha[i,j,k]*phi[i,j,k] - b*h2inv*(beta_i[i+1,j,k] * (phi[i+1,j,k] - phi[i ,j,k]) beta_i[i ,j,k] * (phi[i ,j,k] - phi[i-1,j,k]) + beta_j[i,j+1,k] * (phi[i,j+1,k] - phi[i,j ,k]) beta_j[i,j ,k] * (phi[i,j ,k] - phi[i,j-1,k]) + beta_k[i,j,k+1] * (phi[i,j,k+1] - phi[i,j,k]) beta_k[i,j,k] * (phi[i,j,k] - phi[i,j,k-1]) phi[i,j,k] = phi[i,j,k] **Precomputed from \alpha \& \beta's** lambda[i,j,k] * (helmholtz - rhs[i,j,k])

High-performance baseline implementation...

- construction of the Laplacian, Helmholtz, and GSRB relaxation have been fused to minimize data movement (=2x performance gain).
- Hybrid MPI+OpenMP implementation where everything is threaded

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Code has little reuse and large working sets:

- VC 2nd order => DRAM bandwidth-bound (flop:byte ~ 0.2)
- Cache working set ~350KB per subdomain
- Only 1 stencil sweep per MPI communication step
- Small subdomains = significant % in communication (poor surface:volume)
- Quadruply nested parallelism:
 - no individual loop longer than 64 iterations (bad for OpenMP)
 - worse, 3 loops see exponentially decreasing parallelism





- 2P Cray XE6 node ("Opteron")
- ✤ 2P Intel[®] Xeon[®] E5-2670 ("SNBe")
- ✤ Intel[®] Xeon PhiTM coprocessor ("KNC")

	Opteron	SNBe	KNC ¹
cache per core	64+512KB	32+256KB	32+512KB
cores (threads) per chip	6	8	60 (240)
SIMD-width (DP)	2	4	8
L3 per chip	6MB	20MB	-
chips per node	4	2	1
DP GFlop/s per node	201.6	332.8	1248
STREAM GB/s per node	49.4	70	150
Programming Model	MPI+OMP	MPI+OMP	OpenMP



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	Opteron	SNBe	KNC ¹			
cache per core			32+512KB			
	colored a construction of construction of construction of construction of construction of the construction					
	smeasy to exploit locality					
= easy_to_sha	re data	onchip	-			
chips per node	chips per node 4 2					
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Must generate 8-way SIMD						
to attain pe	ak perf	ormanc	e 1			
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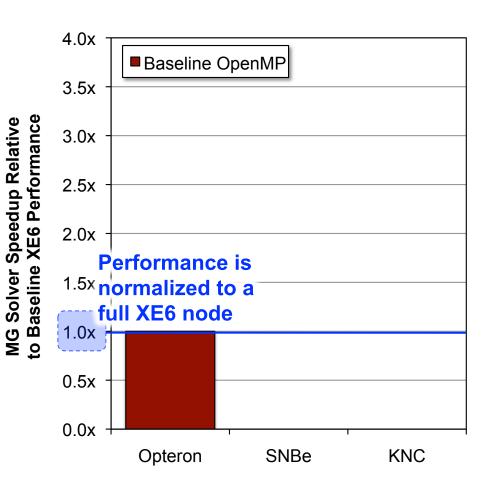
Optimization and Single Node Results

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 We normalize performance (higher is better) to that of a full XE6 (Opteron) node.



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Baseline OpenMP Performance

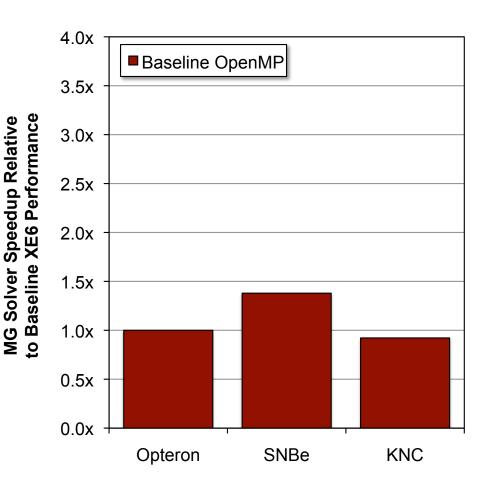
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 Using *native mode* on KNC, we can run exactly the same OpenMP source code on all three platforms (huge productivity win)

.....

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- out-of-the-box KNC performance was less than expected.
- Recall, each loop is no more than 64 iterations (box, i, j, k) and the latter decrease exponentially.
- Baseline code applies a single #pragma omp parallel for to either the box- or k-loop.
- Perfectly acceptable for 8 threads.
- Disastrous approach when there are 240. (73% of the cores on KNC were idle).

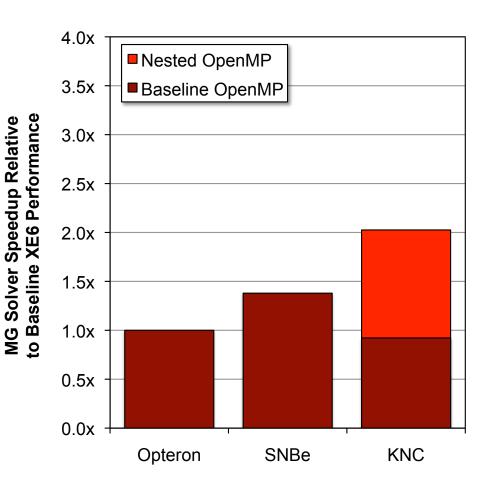




Nested OpenMP Performance

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- Remedy is to apply parallelism hierarchically:
 - some number of threads per box
 - multiple concurrent boxes
- The optimal ratio varies as one descends through the v-cycle.
- Optimal values were determined empirically (auto-tuning)
- Realized within a single OpenMP parallel region with no other optimizations.
- Theoretically, KNC should be twice as fast as SNBe (2x the bandwidth)
 - Iack of SIMD
 - Iack of SW prefetching





An examination of time in conjunction with a performance bound will tell us where to focus our efforts...

		Level in the v-cycle					
operation	64 ³	32 ³	16 ³	8 ³	4 ³	Total	
smooth	63.5%	8.7%	0.4%	0.1%	0.1%	72.8%	
residual	1.1%	1.1%	0.0%	0.0%	0.0%	9.0%	
restriction	1.2%	0.1%	0.0%	0.0%	0.0%	1.3%	
interpolation	1.8%	0.2%	0.0%	0.0%	0.0%	2.0%	
communication	10.7%	2.1%	0.4%	0.2%	0.5%	14.0%	
Total	85.1%	12.2%	0.9%	0.3%	0.6%		



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residual	1.1%	1.1%	0.0%	0.0%	0.0%	9.0%
restriction	2%_0	solve	r _o tim	e is s	pent	1.3%
interpolation	in sm	ooth()	and	resid	u <u>al()</u>	2.0%
communication	10.7%	2.1%	0.4%	0.2%	0.5%	14.0%
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If we construct a **Roofline model** for the solver...

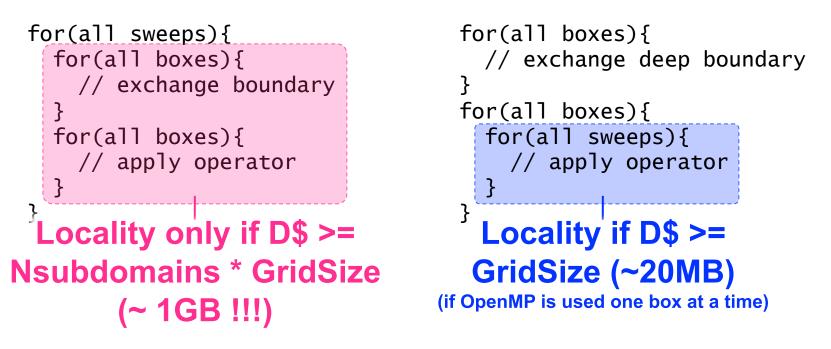
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We're doing the best the SNBe architecture allows.

Only option is to change the algorithm.



- The basic algorithm demands we nominally perform one operation on each box, then communicate (exchange data on/off nodes).
 - destroys any ability to exploit on-chip locality (reuse)
 - incurs repeated overheads
- Leverage the well-known technique of using deeper ghost zones and performing redundant work.

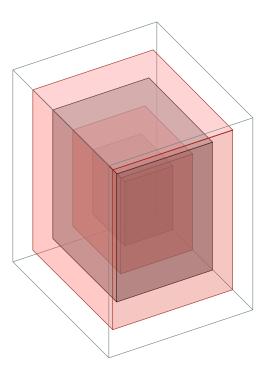




DRAM Communication-Avoiding

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- In theory, a 4-deep ghost zone allows for up to a
 3.1x speedup on smooth on the finest (64³) grids.
- However, in a naïve implementation, one updates a color on the whole grid before the next
 - First (red) sweep is very slow (reading data from DRAM)
 - Next 3 sweeps are fast (reading from LLC) These amortize the first iteration

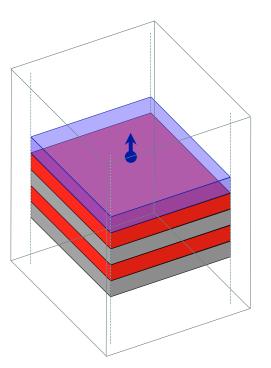




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- ✤ A better solution is to apply it in a wavefront.
 - prefetches (reads) the next plane from DRAM
 - Sequentially apply the GSRB relax to the next four planes (red/black/red/black)
 - Decoupling communication and computation hides the slow DRAM access.

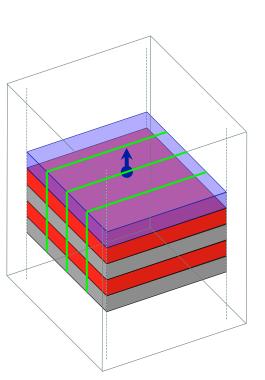




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- Even faster if we thread the wavefront with OpenMP
 - requires cache coherency and fine-grained synchronization (had to write a custom pairwise barrier for performance)



- Explicit SIMDization (via intrinsics)
 - wrote a Perl script to generate the inner AVX and SSE stencil kernels (including SW prefetching). Auto-tuned to find best version.
 - SIMDize GSRB via predication (e.g. blendvpd)
 i.e. all lanes within a SIMD register perform stencils on consecutive elements, but a merge restores the elements not to be modified.
- Fusion of residual and restriction.
 - The residual is immediately restricted (becoming the new RHS)
 - Eliminates the write and re-reading of the residual
- 2MB pages on KNC

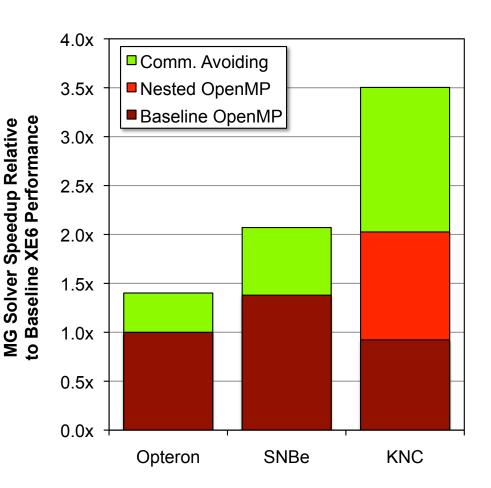
For background papers on auto-tuning, see

http://crd.lbl.gov/about/staff/cds/ftg/samuel-williams



Communication-Avoiding, Fully-Optimized

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- We observe a substantial speedup in overall solver time when both communication-avoiding AND lowlevel optimizations are used...
 - 1.5x on SNBe
 - 1.7x on KNC
- speedup for smooth()...
 - 2.1x on SNBe (out of 3.1x)
 - 2.2x on KNC
- Challenged by large D\$ working set comparable to the L2\$
- Challenged by memory access pattern in smooth (few KB stanza)
 - 66% of STREAM on SNBe
 - 56% of STREAM on KNC





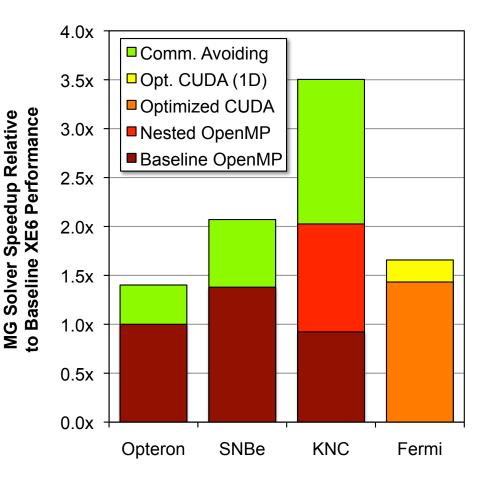
- In the time since the paper submission, we've created a highly-optimized CUDA implementation...
 - parameterized 2D and 1D(=perfect memory coalescing) thread block conceptualizations
 - tuned over subset of several million optimization combinations
- Single-node experiments evaluated using an NVIDIA M2090 (Fermi) GPU with ECC enabled.
- ✤ Machine has a streaming bandwidth of ~120GB/s.



GPU Comparison

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- M2090 is slightly faster than the 2P SNBe on the baseline algorithm
 - M2090 has more bandwidth
 - SNBe likely moves less data
- The portable, nested OpenMP implementation running on KNC is faster than heavily-optimized CUDA on Fermi.
- The combined benefits of processor (KNC), communication-avoiding, and optimization resulted in a 3.5x speedup on the solver time.





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- We have implemented several communication-avoiding versions for the GPU.
- However, they demand we either...
 - have thread blocks perform a great deal of redundant work within a subdomain (bound by SM capabilities)
 - use enormous thread blocks (5K threads and 1.5MB of registers)
 - abandon CUDA ethos and use persistent threads, fine-grained synchronization, and communication through the L2
- In reality...
 - The former has yet to result in better performance than the noncommunication-avoiding implementation
 - The second is a non-starter (we can't change architecture or CUDA), however, the hope is Kepler will mitigate some issues.
 - We have not explored the third.



Summary

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Geometric multigrid solvers for 2nd order operators are heavily bound by data movement

- To improve performance, one must embrace:
 - Faster algorithms that move less data
 - Architectures and DRAM technologies that move data faster

We quantified the benefits of:

- communication-avoiding to address the former
- Use of the Intel[®] Xeon Phi[™] and NVIDIA GPUs to address the latter.



- Xeon PhiTM's use of a coherent cache allows for a simpler (conventional OpenMP) implementations that can minimize data movement whilst exploiting superior GDDR bandwidths
- CUDA/GPUs can constrain one's ability to minimize data movement.
- Manual expression of data parallelism (SIMDization) and data movement (SW prefetching) remained essential on all CPUs.
 - The GPU's SIMT architecture allows one to rewrite code once (in CUDA), and then exploit increased parallelism in future GPUs
 - CPUs must make similar advances in languages/models/compilers.
 - OpenMP needs better constructs/runtimes for hierarchical parallelism
- Even when provided with the opportunity for communicationavoiding (deep ghost zones), compilers utterly fail to exploit it.
 - manually implementing a wavefront is painful and error prone
 - need compilers/DSLs/tools to step up

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- Continued optimization and convergence of CPU, KNC, GPU and (eventually) BGQ implementations
- Distributed studies of KNC and GPUs
 - KNC and GPU were single node/card studies
 - MPI performance can become critical on coarse grids
- High-order relaxation schemes
 - require same deep ghost zones (no more communication)
 - one high arithmetic intensity stencil (instead of multiple passes)
 - no redundant work
- Communication-avoiding bottom solvers
 - scalable, fast bottom solvers demand an altogether different set of optimizations from the v-cycle



- All authors from Lawrence Berkeley National Laboratory were supported by the DOE Office of Advanced Scientific Computing Research under contract number DE-AC02-05CH11231.
- This research used resources of the National Energy Research Scientific Computing Center, which is supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231.
- This work was supported in part by National Science Foundation grants: OCI #0910847, Gordon: A Data Intensive Supercomputer; and OCI #1053575, Extreme Science and Engineering Discovery Environment (XSEDE).
- This research used resources of the Keeneland Computing Facility at the Georgia Institute of Technology, which is supported by the National Science Foundation under Contract OCI-0910735.



Questions?

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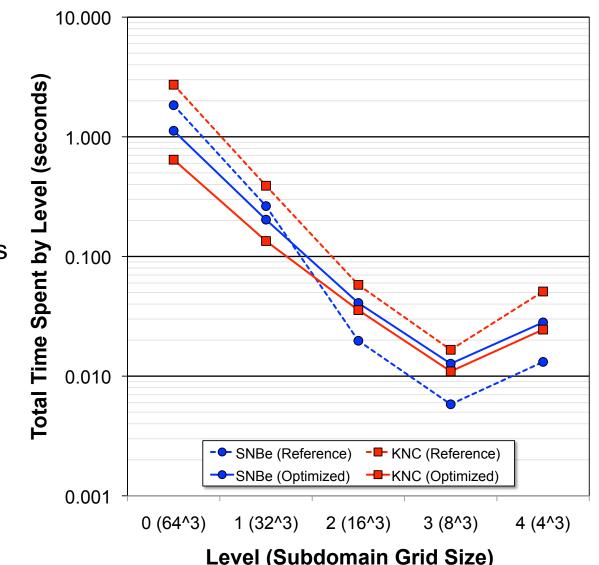
Backup Slides



Time per Level

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- Optimized KNC was fastest at the finest levels.
- Eventually, there is a crossover, and unoptimized (no CA) was fastest at the bottom (all subdomains fit in LLC)



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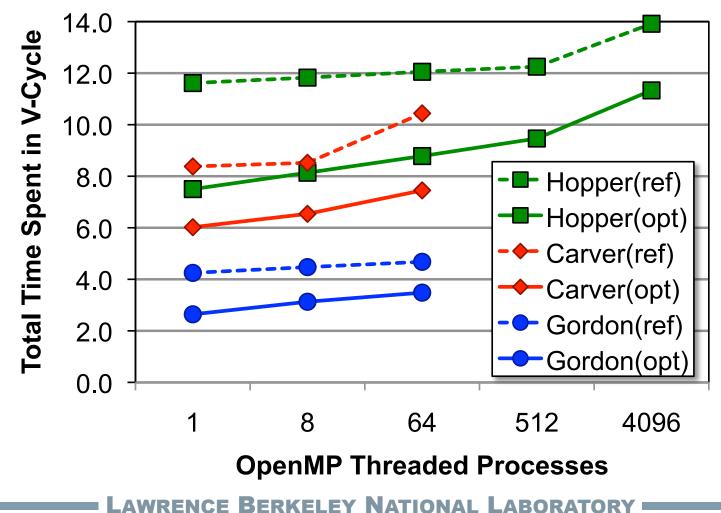
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Weak Scaling of the V-cycle

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- ✤ 256³ (64 x 64³ subdomains) per NUMA node
- 1 'threaded process' = 1 chip





* ...

NVIDIA M2090 ("Fermi")

	Opteron	SNBe	KNC ¹	Fermi ²
cache per core	64+512KB	32+256KB	32+512KB	192KB
cores (threads) per chip	6	8	60 (240)	16
SIMD-width (DP)	2	4	8	32
LLC per chip	6MB	20MB	-	768KB
chips per node	4	2	1	1
DP GFlop/s per node	201.6	332.8	1248	665
STREAM GB/s per node	49.4	70	150	120
Programming Model	MPI+OMP	MPI+OMP	OpenMP	CUDA

¹Evaluation card is not necessarily reflective of production card specifications ²We consider 1 SM as the equivalent of 1 CPU core



1D CUDA Thread Blocks

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- ✤ Within a block, there are...
 - Threads that will perform the stencil
 - Threads to hold the halo (within the plane)
 - Threads to guarantee coalescing
- Thread block streams thru the k-dimension
- Can tune for any # of threads (restricted to multiples of 16 or 32)
- Optimal # of threads/block were 480, 512, or 992

