

# The Roofline Model:

A pedagogical tool for program analysis and optimization

#### ParLab Summer Retreat

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#### **Motivation**



- Performance and scalability of multicore architectures can be extremely non-intuitive to novice programmers
- Success of the multicore paradigm should be premised on augmenting the abilities of the world's programmers



#### **Goals & Audience**



#### Focused on:

rates and efficiencies (Gflop/s, % of peak),

- Goals for Roofline:
  - Provide everyone with a graphical aid that provides:
     realistic expectations of performance and productivity
  - Show inherent hardware limitations for a given kernel
  - Show potential benefit and priority of optimizations
- Who's not the audience for the Roofline:
  - Not for those interested in fine tuning (+5%)
  - Not for those challenged by parallel kernel correctness



# Principal Components of Performance



#### Components



- There are three principal components to performance:
  - Computation
  - Communication
  - Locality
- Each architecture has a different balance between these
- Each kernel has a different balance between these
- Performance is a question of how well an kernel's characteristics map to an architecture's characteristics



### Computation



- For us, floating point performance (Gflop/s) is the metric of interest (typically double precision)
- Peak in-core performance can only be attained if:
  - fully exploit ILP, DLP, FMA, etc...
  - non-FP instructions don't sap instruction bandwidth
  - threads don't diverge (GPUs)
  - transcendental/non pipelined instructions are used sparingly
  - branch mispredictions are rare
- To exploit a form of in-core parallelism, it must be:
  - Inherent in the algorithm
  - Expressed in the high level implementation
  - Explicit in the generated code



#### Communication



- For us, DRAM bandwidth (GB/s) is the metric of interest
- Peak bandwidth can only be attained if certain optimizations are employed:
  - Few unit stride streams
  - NUMA allocation and usage
  - SW Prefetching
  - Memory Coalescing (GPU)



### Locality



- Computation is free, Communication is expensive.
- Maximize locality to minimize communication
- There is a lower limit to communication: compulsory traffic
- Hardware changes can help minimize communication
  - Larger cache capacities minimize capacity misses
  - Higher cache associativities minimize conflict misses
  - Non-allocating caches minimize compulsory traffic
- Software optimization can also help minimize communication
  - Padding avoids conflict misses
  - Blocking avoids capacity misses
  - Non-allocating stores minimize compulsory traffic





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# **Roofline Model**



## Integrating Components...



- Goal: integrate in-core performance, memory bandwidth, and locality into a single readily understandable performance figure
- Also, must graphically show the penalty associated with not including certain software optimizations

- Roofline model will be unique to each architecture
- Coordinates of a kernel are ~unique to each architecture



## What relates GB/s to GFlop/s?



- Through dimensional analysis, its clear that Flops:Bytes is the parameter that allows us to convert bandwidth (GB/s) to performance (GFlop/s)
- This is a well known quantity: Arithmetic Intensity (discussed later)
- When we measure total bytes, we incorporate all cache behavior (the 3C's) and Locality



#### **Basic Roofline**



Performance is upper bounded by both the peak flop rate, and the product of streaming bandwidth and the flop:byte ratio



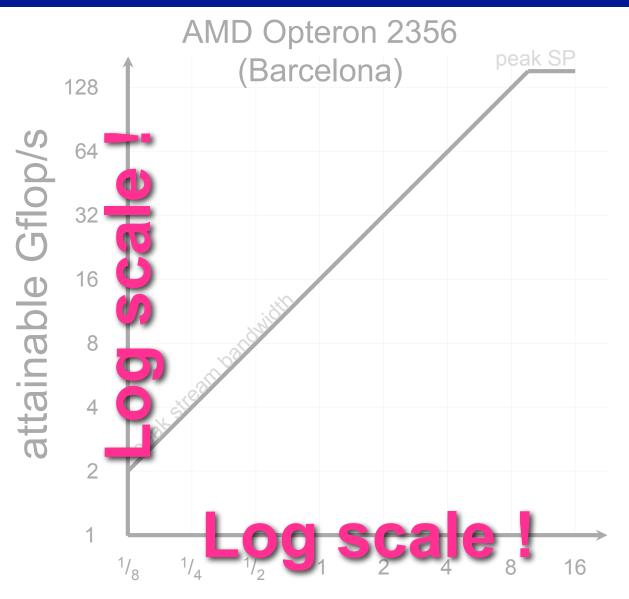
#### notes



- Bandwidth #'s collected via micro benchmarks
- Computation #'s derived from optimization manuals (pencil and paper)
- Assume complete overlap of either communication or computation



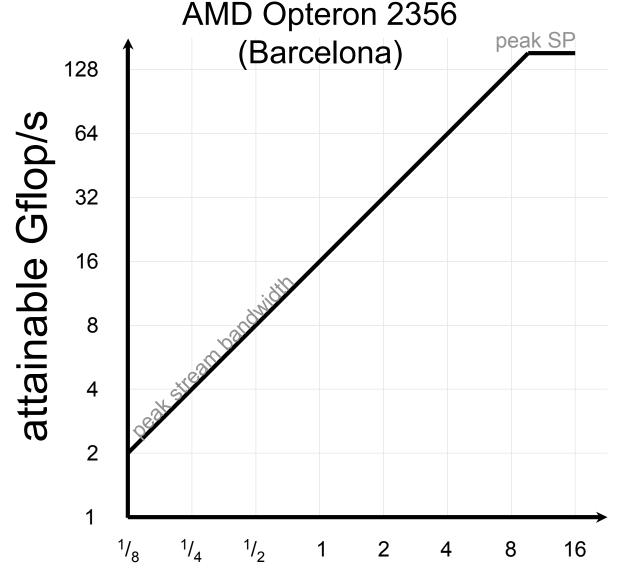




- Peak roofline performance
- based on manual for single precision peak
- and a hand tuned stream read for bandwidth



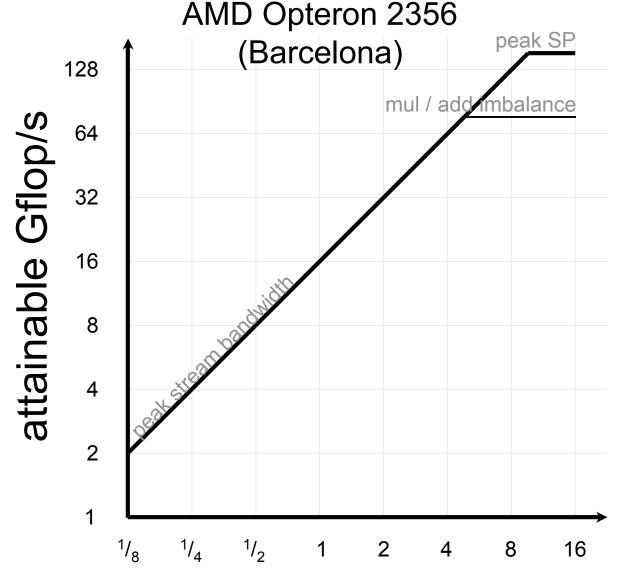




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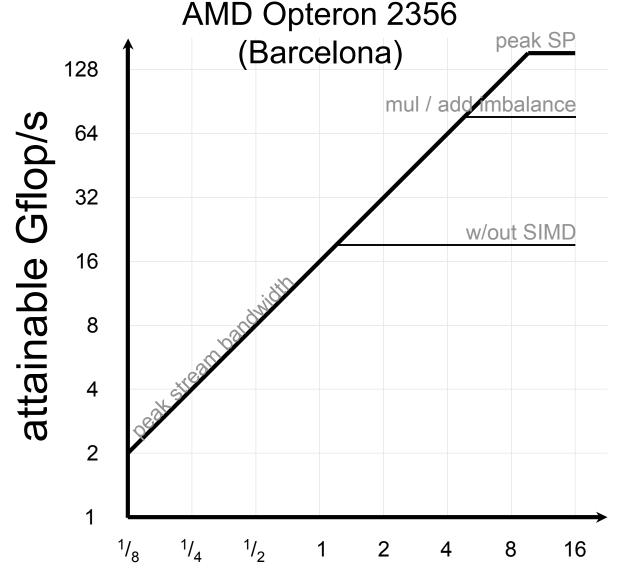




- Opterons have separate multipliers and adders
- 'functional unit parallelism'
- This is a ceiling beneth the roofline





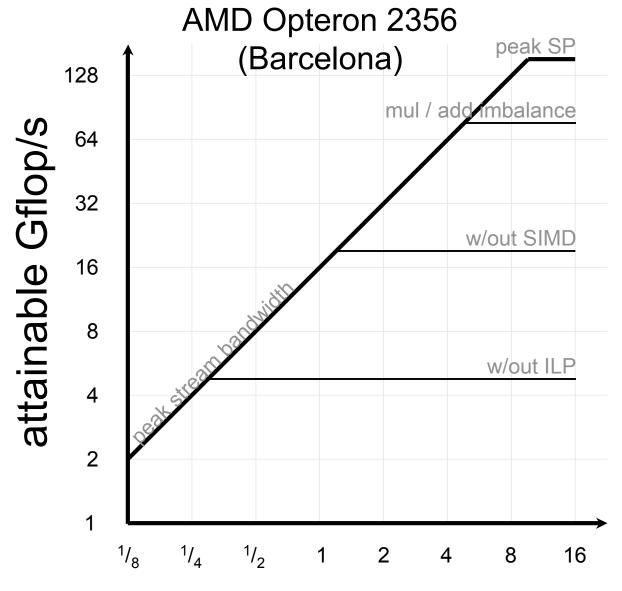


- In single precision, SIMD is 4x32b.
- If only the \_ss versions are used, performance is 1/4





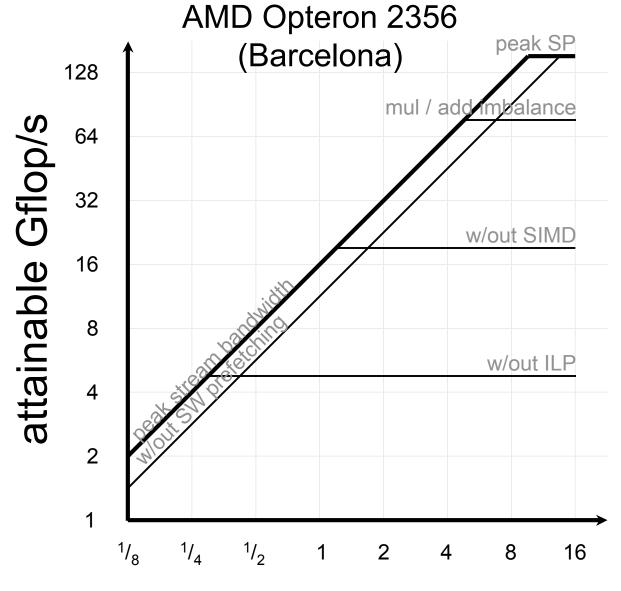
(adding ceilings)



If 4 independent instructions are kept in the pipeline, performance will fall





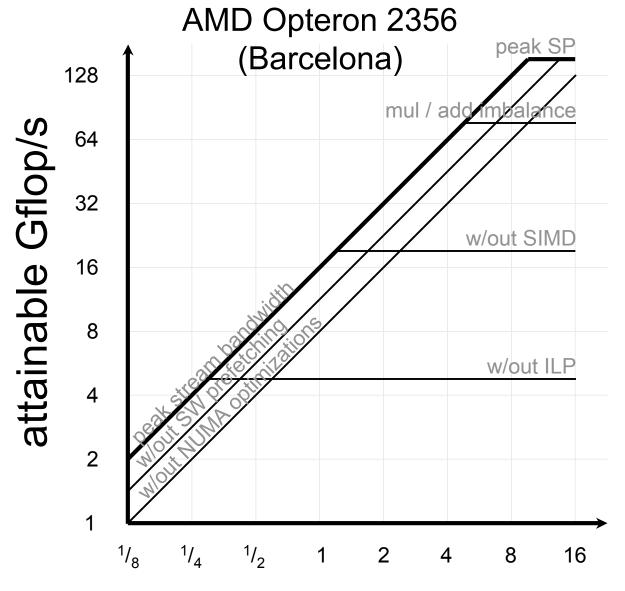


- If SW prefetching is not used, performance will degrade
- These act as ceilings below the bandwidth roofline





(adding ceilings)

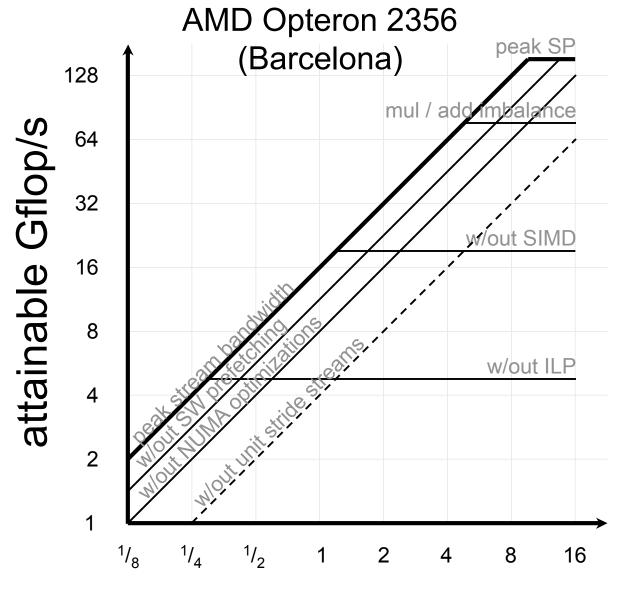


Without NUMA optimizations, the memory controllers on the second socket can't be used.





(adding ceilings)

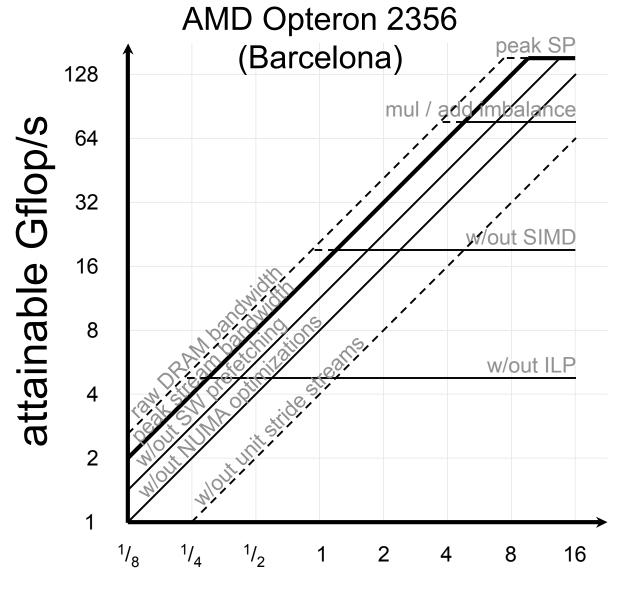


Bandwidth is much lower without unit stride streams





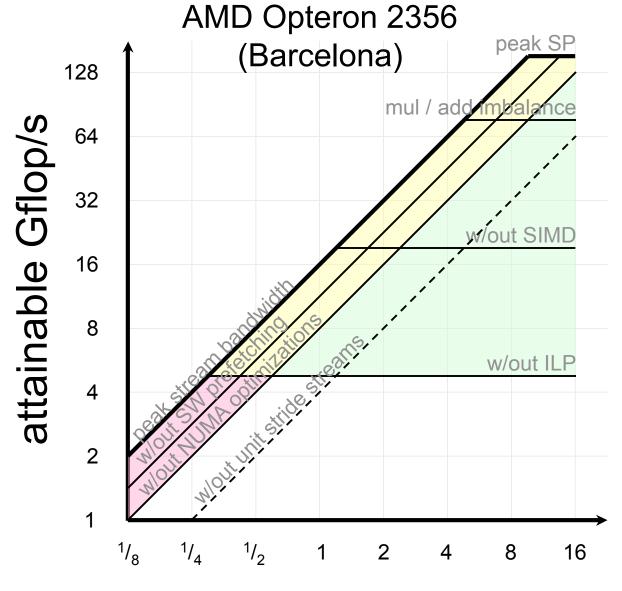
(adding ceilings)



 Its difficult for any architecture to reach the raw DRAM bandwidth







- Partitions the regions of expected performance into three optimization regions:
  - Compute only
  - Memory only
  - Compute+Memory



#### Uniqueness

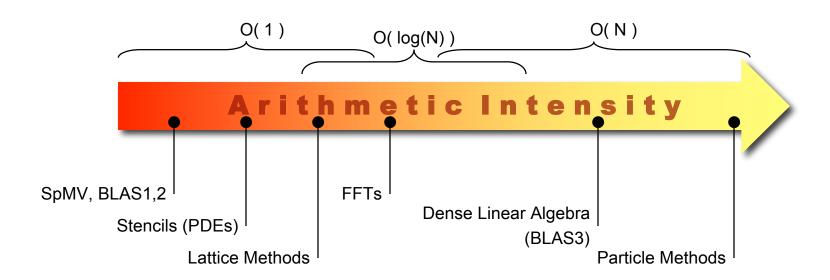


- There is no single ordering or roofline model
- The order of ceilings is generally (bottom up):
  - What is inherent in algorithm
  - What a compiler is likely to provide
  - What a programmer could provide
  - What can never be exploited for this kernel
- For example,
  - FMA or mul/add balance is inherent in many linear algebra routines and should be placed at the bottom.
  - However, many stencils are dominated by adds, and thus the multipliers and FMA go underutilized.



#### **Arithmetic Intensity in HPC**





- ❖ Arithmetic Intensity (AI) ~ Total Flops / Total DRAM Bytes
- Some HPC kernels have an arithmetic intensity that's constant, but on others it scales with with problem size (increasing temporal locality)
- Actual arithmetic intensity is capped by cache/local store capacity



# Accurately Determining the true Flop:DRAM Byte ratio

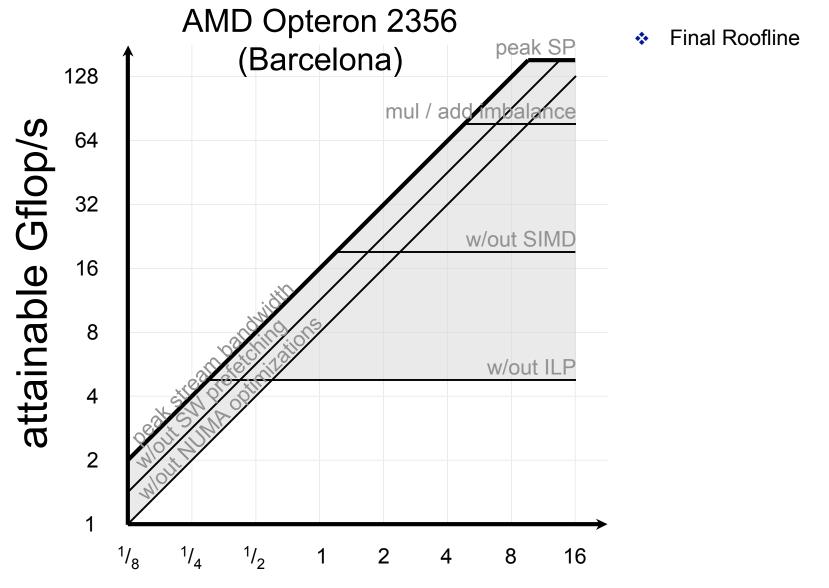


- Remember the 3C's of caches
- Calculating the Flop:DRAM byte ratio is:
  - Compulsory misses: straightforward
  - Capacity misses: pencil and paper (maybe performance counters)
  - Conflict misses: must use performance counters

- Flop:actual DRAM Byte ratio < Flop:compulsory DRAM Byte ratio</p>
- One might place a range on the arithmetic intensity ratio
- Thus performance is limited to an area between the ceilings and between the upper (compulsory) and lower bounds on arithmetic intensity

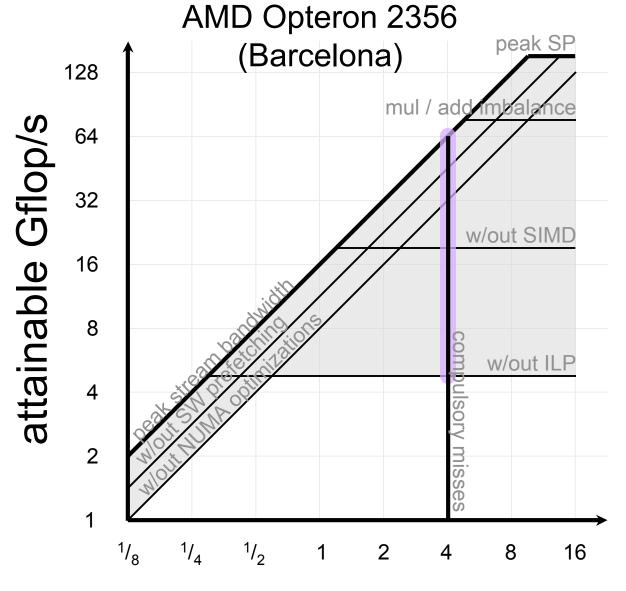








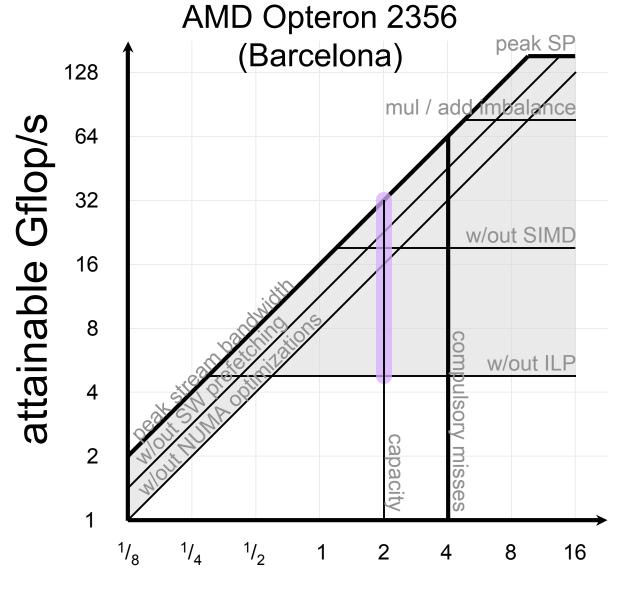




- Some arbitrary kernel has a flop:compulsory byte ratio of 4
- Overlaid on the roofline
- Defines upper bound on range of expected performance
- Also shows which optimizations are likely



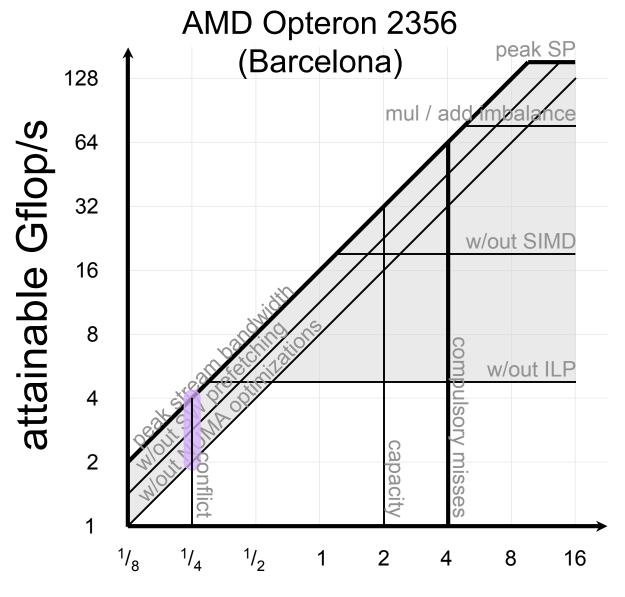




- Capacity misses reduce the actual flop:byte ratio
- Also reduces attainable performance
- Al is unique to each combination of kernel and architecture





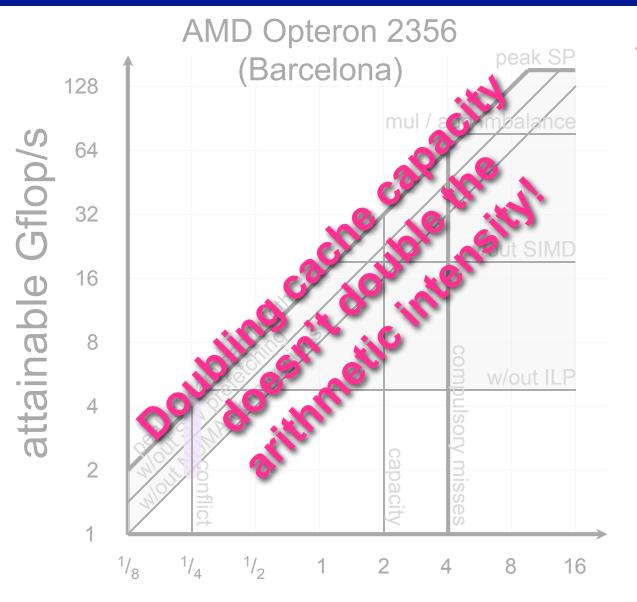


- Conflict misses may destroy performance
- Al is unique to each combination of kernel and architecture





(powerpoint doodle)



 Conflict misses may destroy performance





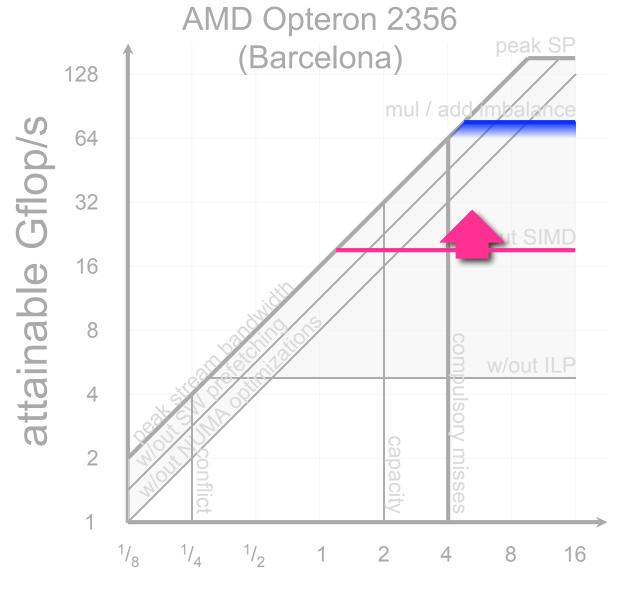
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# Three Categories of Software Optimization



# Maximizing Attained in-core Performance



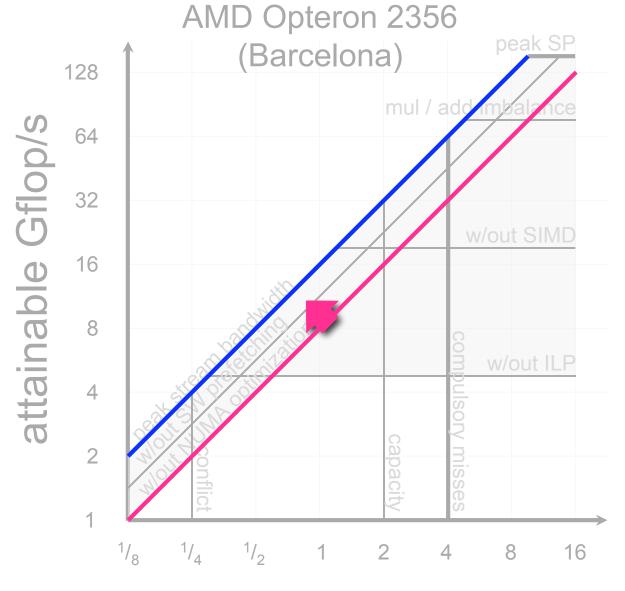


- Software optimizations such as explicit
   SIMDization can punch through the horizontal ceilings (what can be expected from a compiler)
- Other examples include loop unrolling, reordering, and long running loops



# Maximizing Attained Memory Bandwidth



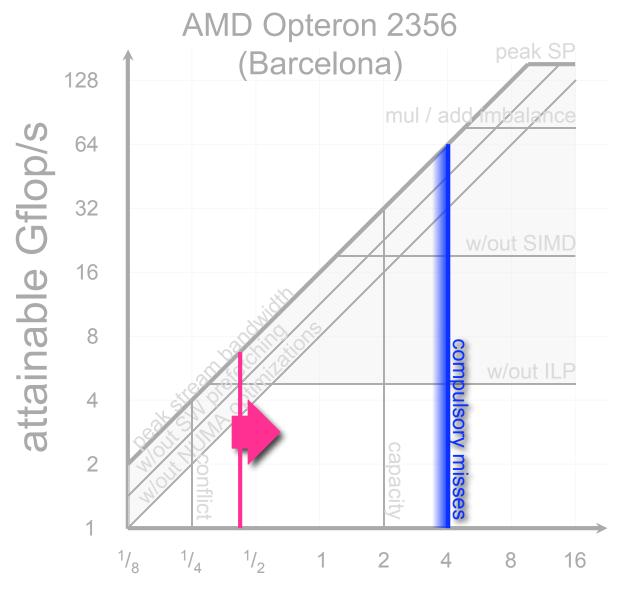


- Compilers won't give great out-of-the box bandwidth
- Punch through bandwidth ceilings:
  - Maximize MLP
  - long unit stride accesses
  - NUMA aware allocation and parallelization
  - SW prefetching



## Minimizing Memory Traffic



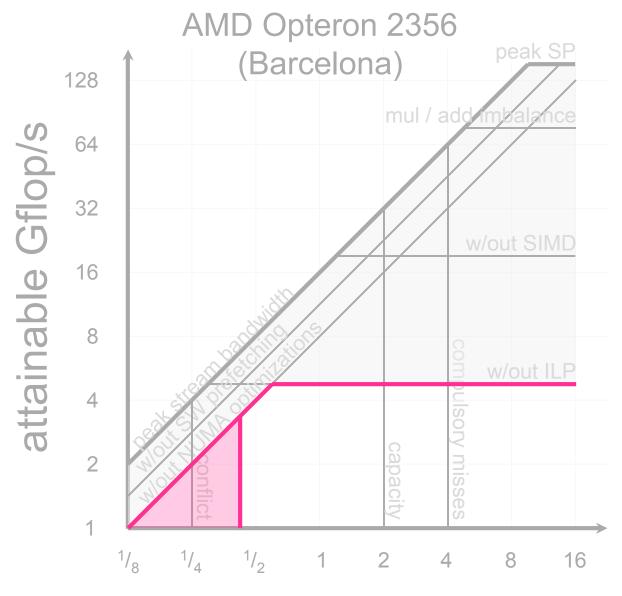


- Use performance counters to measure flop:byte ratio (AI)
- Out-of-the-box code may have an AI ratio much less than the compulsory ratio
  - Be cognizant of cache capacities, associativities, and threads sharing it
  - Pad structures to avoid conflict misses
  - Use cache blocking to avoid capacity misses
- These optimizations can be imperative



## **Effective Roofline (before)**



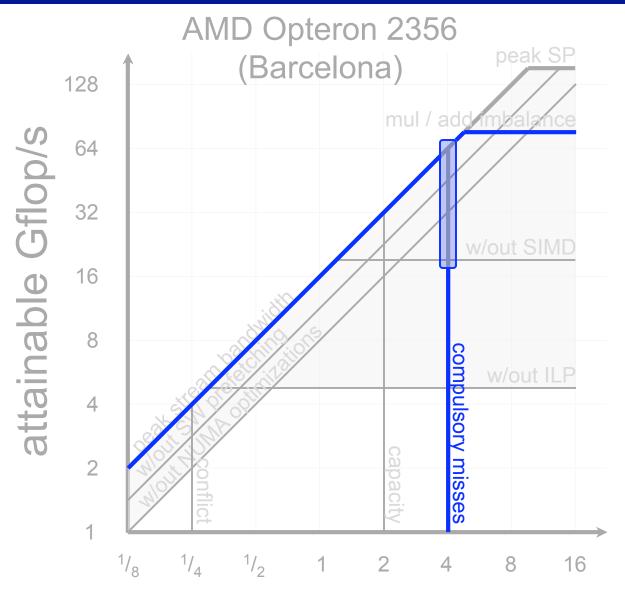


 Before optimization, traffic, and limited bandwidth optimization limits performance to a very narrow window



## **Effective Roofline (after)**





 After optimization, ideally, performance is significantly better





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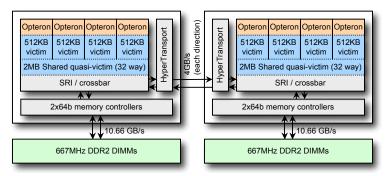
# Applicable to Other Architectural Paradigms?



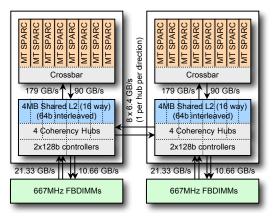
#### **Four Architectures**



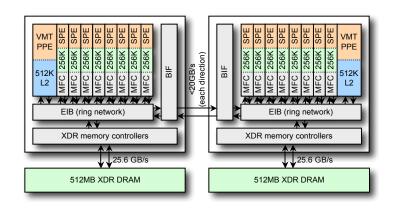
#### AMD Barcelona



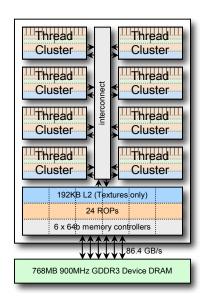
#### Sun Victoria Falls



#### IBM Cell Blade



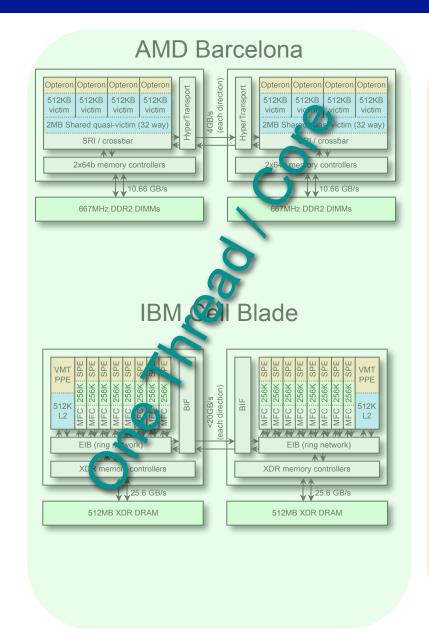
#### **NVIDIA G80**

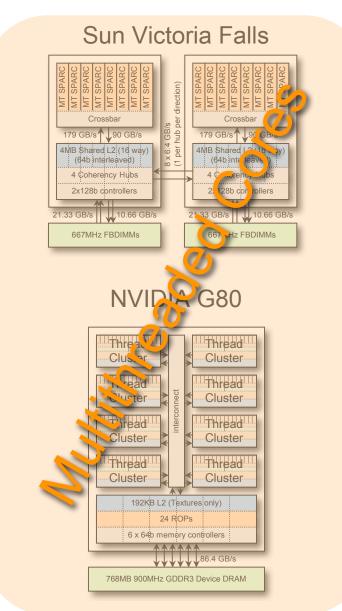




#### **Four Architectures**



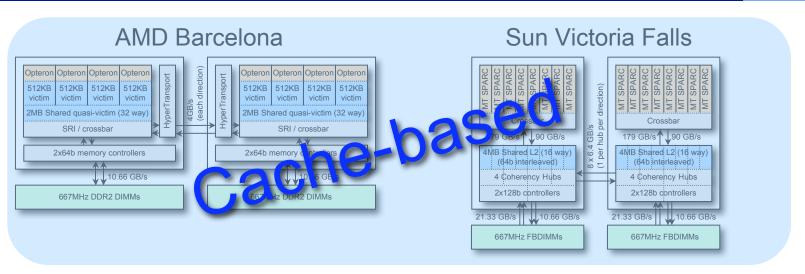






#### **Four Architectures**



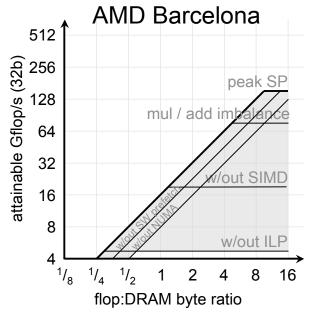


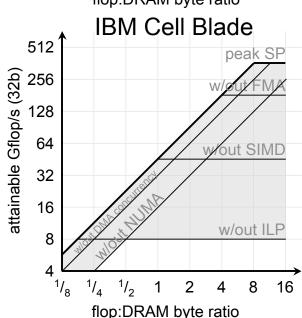


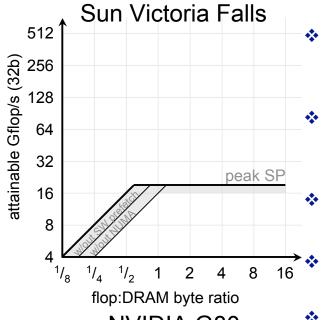


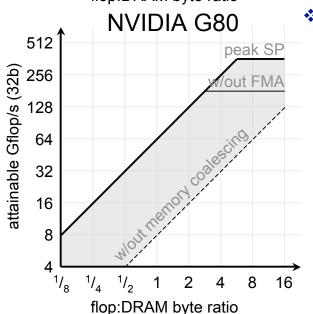


(in-core parallelism)









Single Precision Roofline models for the SMPs used in this work.

Based on microbenchmarks, experience, and manuals

Ceilings =

in-core parallelism

Can the compiler find all this parallelism?

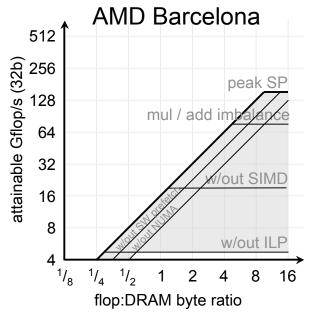
#### NOTE:

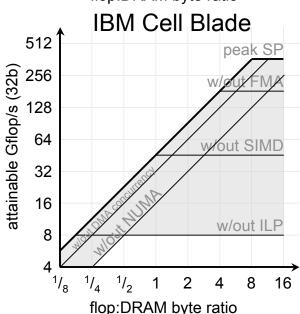
- log-log scale
- Assumes perfect SPMD

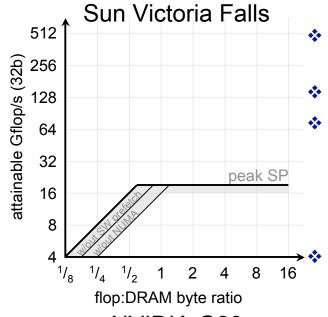


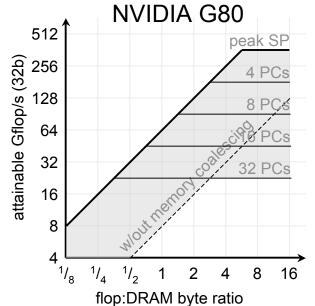


(diverged threads)









G80 dynamically finds DLP (shared instruction fetch)

#### SIMT

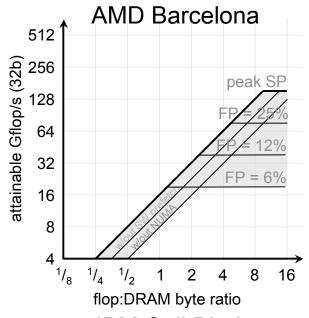
If threads of a warp diverge from SIMD execution, performance is limited by instruction issue bandwidth

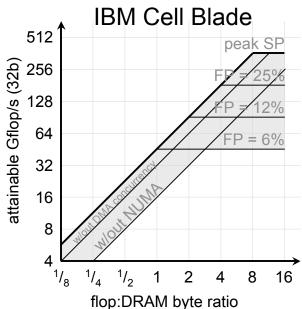
Ceilings on G80 = number of unique PCs when threads diverge

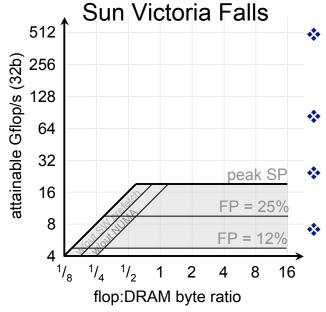


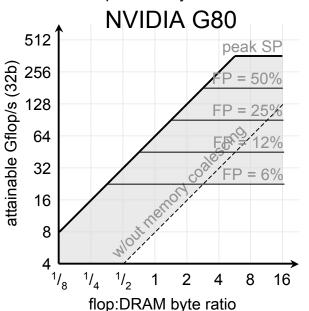


(FP fraction of dynamic instructions)









Some kernels have large numbers of non FP instructions

Saps instruction issue bandwidth

Ceilings = FP fraction of dynamic instruction mix

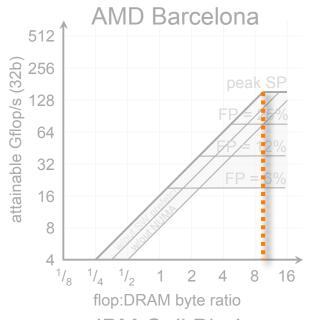
#### NOTE:

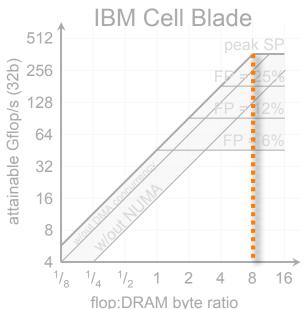
 Assumes perfect in-core parallelism

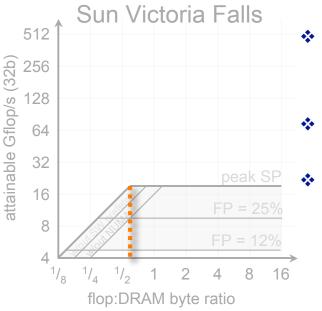


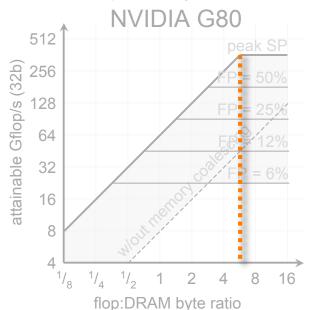


(ridge point)









Some architectures have drastically different ridge points

VF may be compute bound on many kernels

Clovertown has  $\frac{1}{3}$  the BW of Barcelona = ridge point to the right





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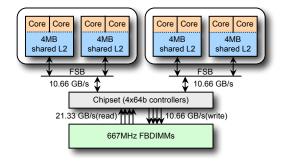
## Using Roofline when Auto-tuning HPC Kernels



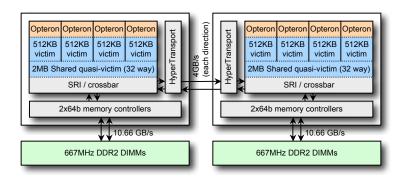
#### **Multicore SMPs Used**



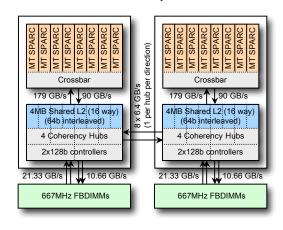
#### Intel Xeon E5345 (Clovertown)



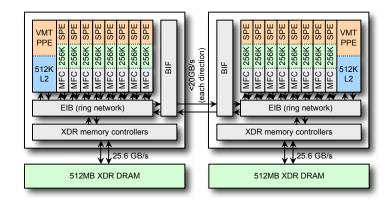
#### AMD Opteron 2356 (Barcelona)



#### Sun T2+ T5140 (Victoria Falls)



#### IBM QS20 Cell Blade







# **Sparse Matrix-Vector Multiplication (SpMV)**

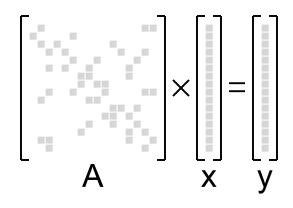
Samuel Williams, Leonid Oliker, Richard Vuduc, John Shalf, Katherine Yelick, James Demmel, "Optimization of Sparse Matrix-Vector Multiplication on Emerging Multicore Platforms", Supercomputing (SC), 2007.



## Sparse Matrix Vector Multiplication

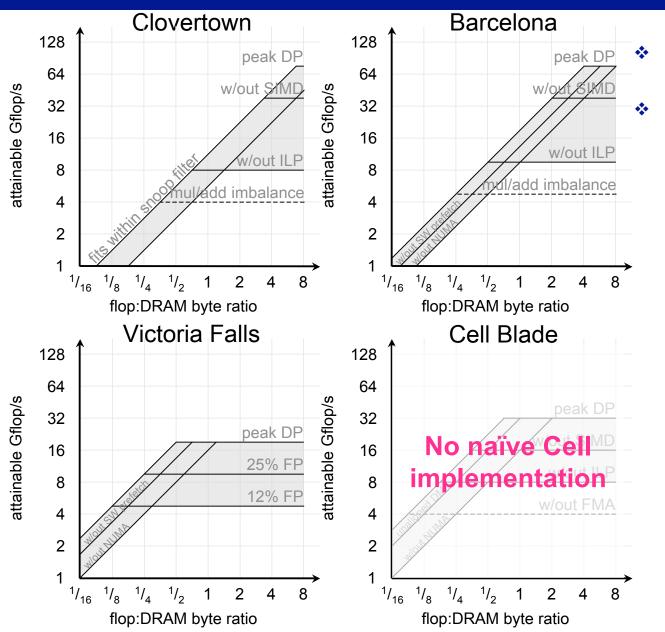


- Sparse Matrix
  - Most entries are 0.0
  - Performance advantage in only storing/operating on the nonzeros
  - Requires significant meta data
- Evaluate y=Ax
  - A is a sparse matrix
  - x & y are dense vectors
- Challenges
  - Difficult to exploit ILP(bad for superscalar),
  - Difficult to exploit DLP(bad for SIMD)
  - Irregular memory access to source vector
  - Difficult to load balance
  - Very low arithmetic intensity (often <0.166 flops/byte)</li>
    - = likely memory bound







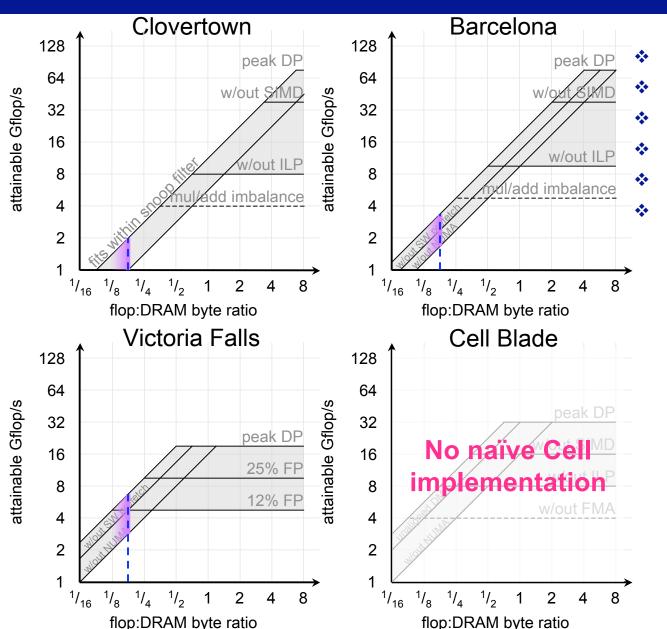


Double precision roofline models

FMA is inherent in SpMV (place at bottom)







Two unit stride streams Inherent FMA No ILP

No DLP

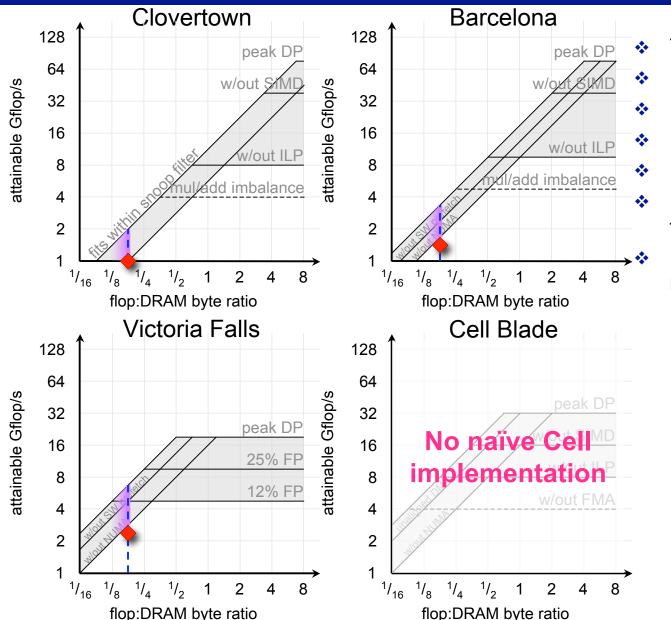
FP is 12-25%

Naïve compulsory flop:byte < 0.166





(out-of-the-box parallel)



Two unit stride streams
Inherent FMA
No ILP
No DLP
FP is 12-25%

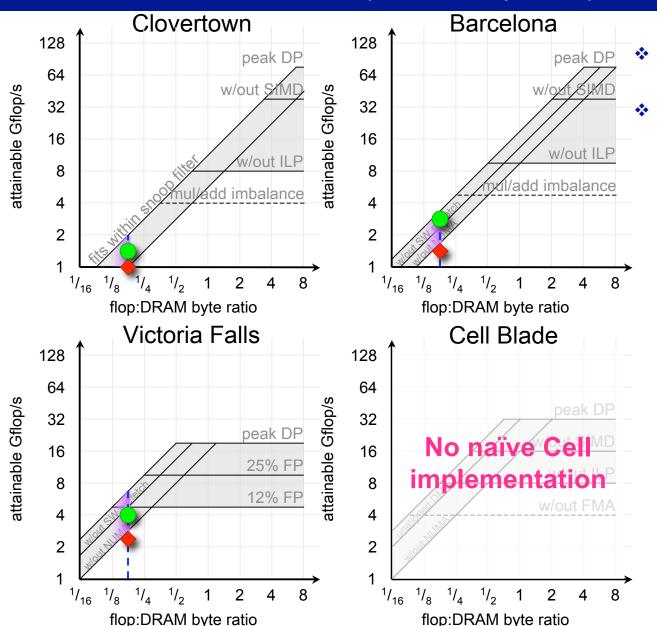
Naïve compulsory flop:byte < 0.166

For simplicity: dense matrix in sparse format





(NUMA & SW prefetch)



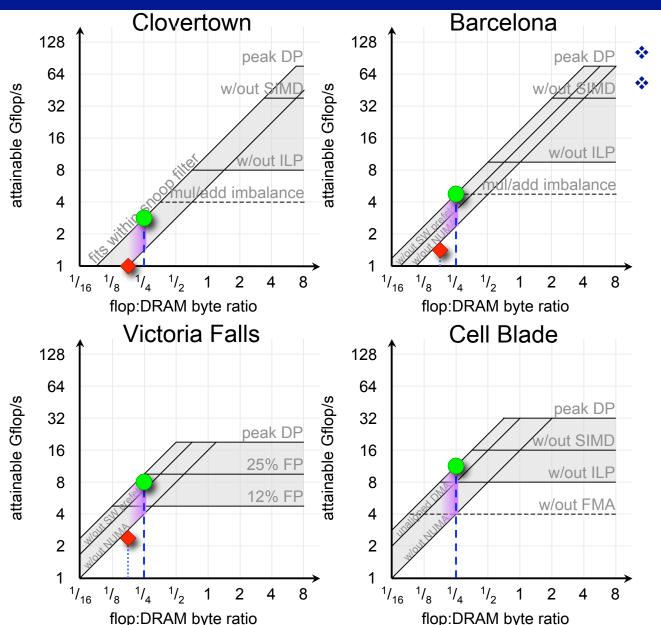
compulsory flop:byte ~ 0.166

utilize all memory channels





(matrix compression)



Inherent FMA

Register blocking improves ILP, DLP, flop:byte ratio, and FP% of instructions



## Lattice-Boltzmann Magneto-Hydrodynamics (LBMHD)

Samuel Williams, Jonathan Carter, Leonid Oliker, John Shalf, Katherine Yelick, "Lattice Boltzmann Simulation Optimization on Leading Multicore Platforms", International Parallel & Distributed Processing Symposium (IPDPS), 2008.

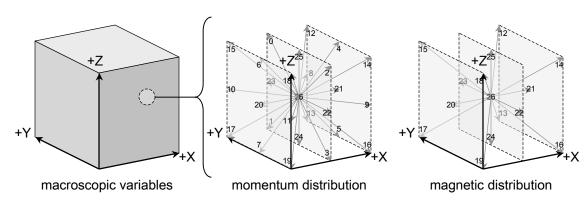
**Best Paper, Application Track** 



#### **LBMHD**



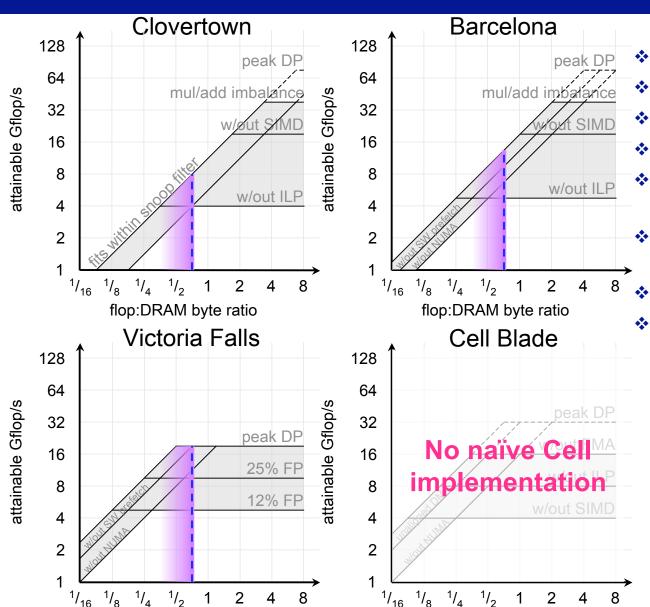
- Plasma turbulence simulation via Lattice Boltzmann Method
- Two distributions:
  - momentum distribution (27 scalar components)
  - magnetic distribution (15 vector components)
- Three macroscopic quantities:
  - Density
  - Momentum (vector)
  - Magnetic Field (vector)
- Must read 73 doubles, and update 79 doubles per point in space
- Requires about 1300 floating point operations per point in space
- Just over 1.0 flops/byte (ideal)





flop:DRAM byte ratio





flop:DRAM byte ratio

Huge datasets
NUMA allocation/access
Little ILP

No DLP

Far more adds than multiplies (imbalance)

**Essentially random** access to memory

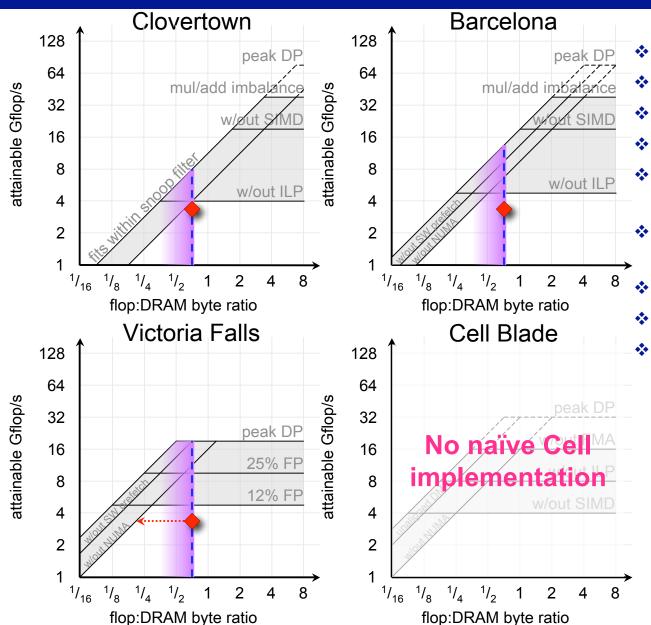
Flop:byte ratio ~0.7

High conflict misses





(out-of-the-box code)



Huge datasets
NUMA allocation/access

Little ILP

No DLP

Far more adds than multiplies (imbalance)

**Essentially random** access to memory

Flop:byte ratio ~0.7

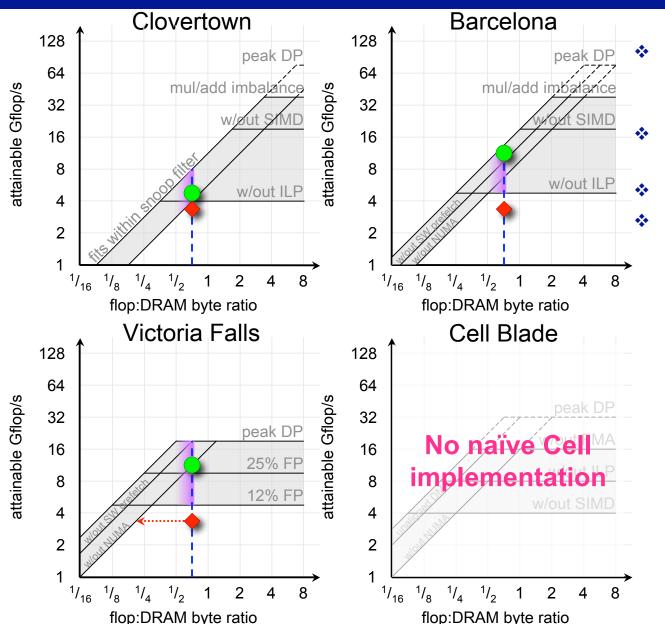
High conflict misses

Peak VF performance with 64 threads (our of 128) - high conflict misses





(Padding, Vectorization, Unrolling, Reordering)



Vectorize the code to eliminate TLB capacity misses

Ensures unit stride access (bottom bandwidth ceiling)

Tune for optimal VL

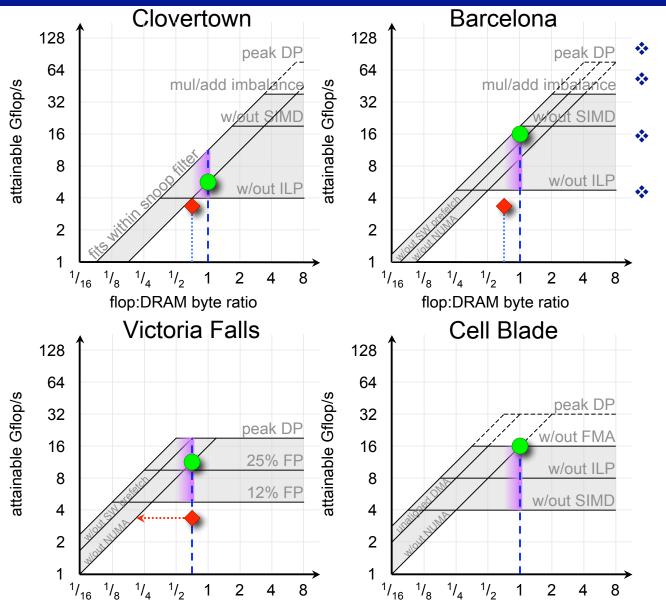
Clovertown pinned to lower BW ceiling





(SIMDization + cache bypass)

flop:DRAM byte ratio



flop:DRAM byte ratio

Make SIMDization explicit
Technically, this swaps ILP
and SIMD ceilings
Use cache bypass
instruction: *movntpd* 

Increases flop:byte ratio to ~1.0 on x86/Cell



## The Heat Equation Stencil

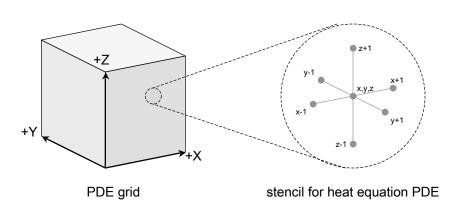
Kaushik Datta, Mark Murphy, Vasily Volkov, Samuel Williams, Jonathan Carter, Leonid Oliker, David Patterson, John Shalf, Katherine Yelick, "Stencil Computation Optimization and Autotuning on State-of-the-Art Multicore Architecture", submitted to Supercomputing (SC), 2008.



## The Heat Equation Stencil



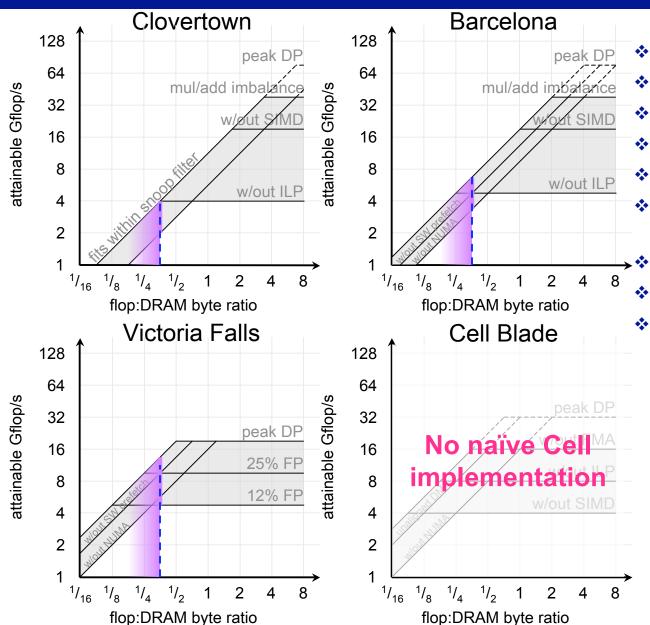
- Explicit Heat equation on a regular grid
- Jacobi
- One double per point in space
- 7-point nearest neighbor stencil
- Must:
  - read every point from DRAM
  - perform 8 flops (linear combination)
  - write every point back to DRAM
- Just over 0.5 flops/byte (ideal)
- Cache locality is important







(out-of-the-box code)



Large datasets

2 unit stride streams

No NUMA

Little ILP

No DLP

Far more adds than multiplies (imbalance)

Ideal flop:byte ratio <sup>1</sup>/<sub>3</sub>

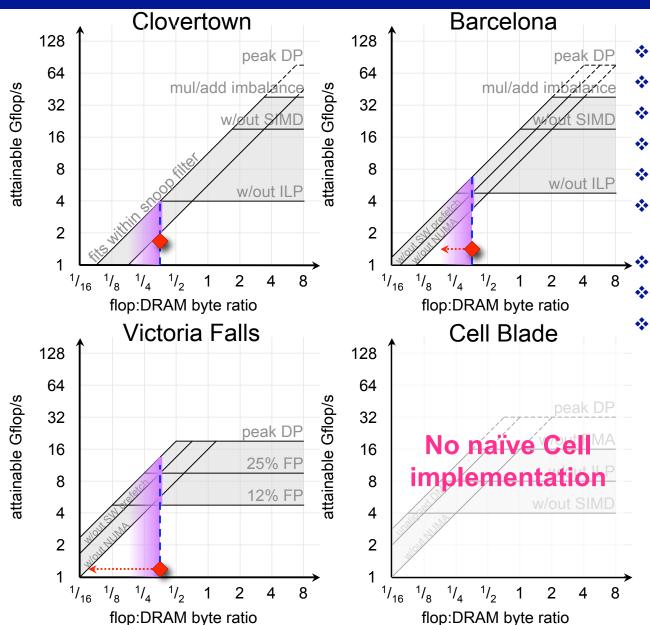
High locality requirements

Capacity and conflict misses will severely impair flop:byte ratio





(out-of-the-box code)



Large datasets

2 unit stride streams

No NUMA

Little ILP

No DLP

Far more adds than multiplies (imbalance)

Ideal flop:byte ratio <sup>1</sup>/<sub>3</sub>

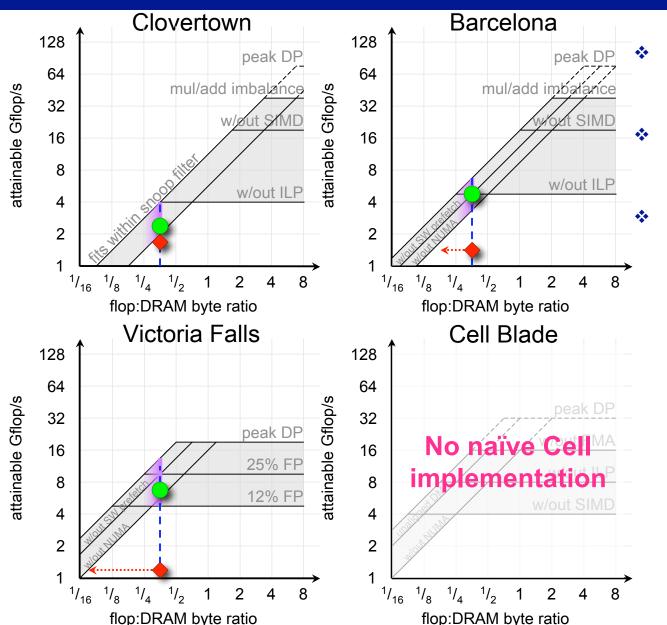
High locality requirements

Capacity and conflict misses will severely impair flop:byte ratio





(NUMA, cache blocking, unrolling, prefetch, ...)



Cache blocking helps ensure flop:byte ratio is as close as possible to  $^{1}/_{3}$  Clovertown has huge caches but is pinned to

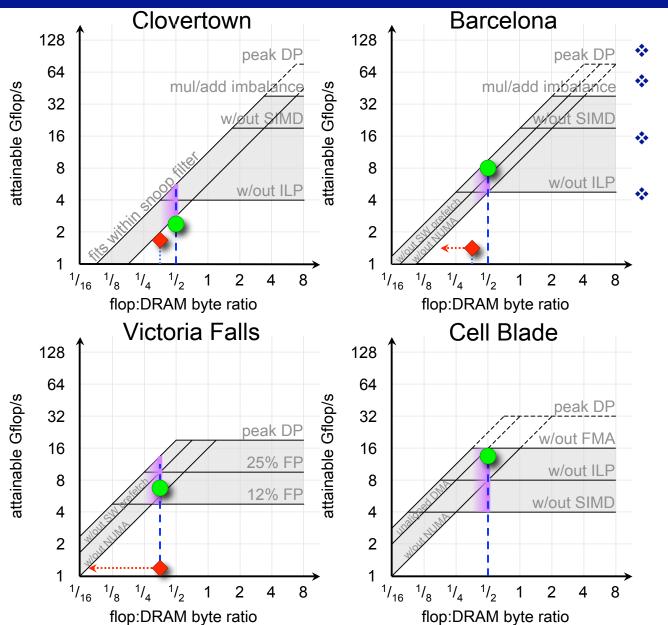
Cache management is essential when capacity/thread is low

lower BW ceiling





(SIMDization + cache bypass)



Make SIMDization explicit
Technically, this swaps ILP
and SIMD ceilings
Use cache bypass
instruction: *movntpd*Increases flop:byte ratio to

~0.5 on x86/Cell





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## Refining the Roofline



#### Other performance metrics



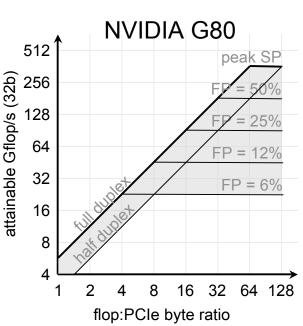
- There is no reason either floating point (Gflop/s) must be the performance metric
- Could also use:
  - Graphics (Pixels, Vertices, Textures)
  - Crypto
  - Integer
  - Bitwise
  - etc...



#### Other bandwidths



- For our kernels, DRAM bandwidth is the key communication component.
- For other kernels, other bandwidths might be more appropriate
  - L2 bandwidth (e.g. DGEMM)
  - PCIe bandwidth (offload to GPU)
  - Network bandwidth
- The example below shows zero overhead double buffered transfers to/from a GPU over PCIe x16
  - How bad is a SP stencil?
  - What about SGEMM?
- No overlap / high overhead tends to smooth performance
  - Performance is half at ridge point





#### Mix and match



- In general, you can mix and match as the kernel/architecture requires:
- e.g. all posibilities is the cross product of performance metrics with bandwidths

{Gflop/s, GIPS, crypto, ...} × {L2, DRAM, PCIe, Network}



#### **Conclusions**



- The Roofline Model provides an intuitive graph for kernel analysis and optimization
- Easily extendable to other architectural paradigms
- Easily extendable to other communication or computation metrics





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## **Questions?**



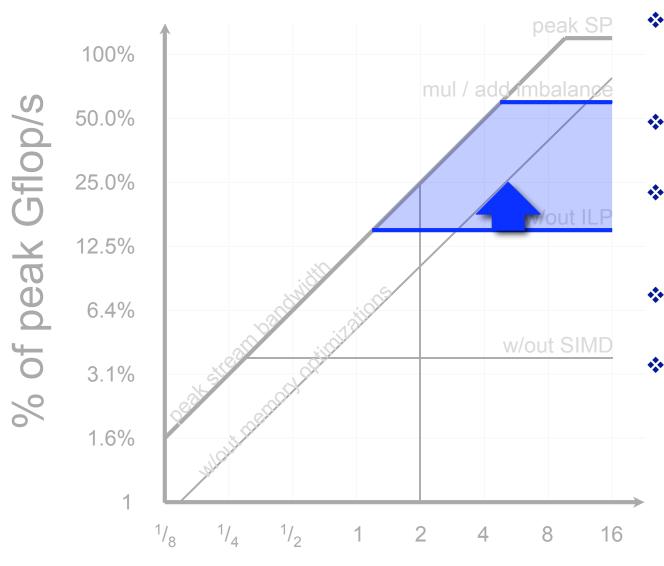


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## **BACKUP SLIDES**



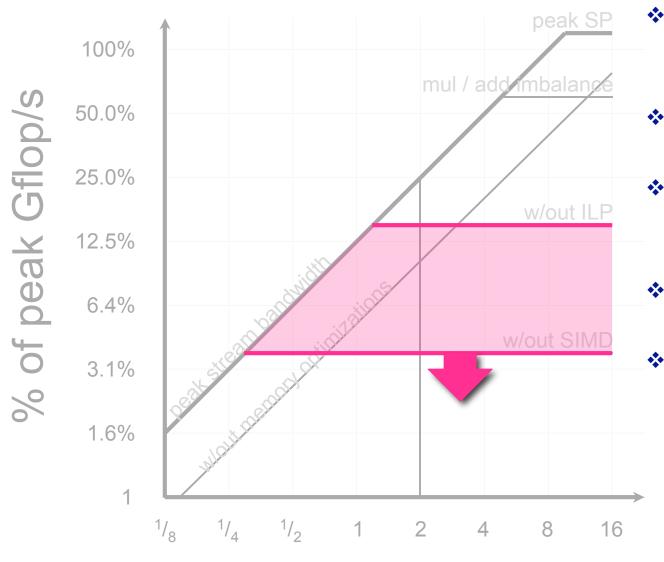




- ILP is decreasing (shorter pipelines, multithreading)
- SIMD is becoming wider
- Bandwidth isn't keeping up with #cores
- Application flop:byte is decreasing
  - Cache/Local Store management is becoming critical



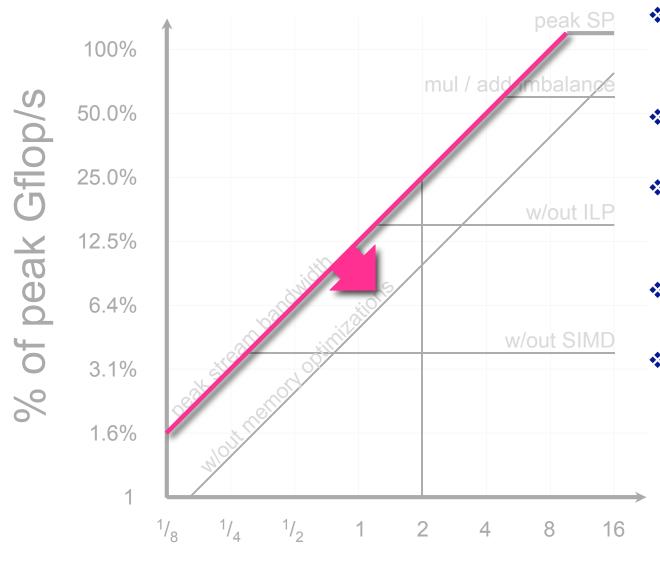




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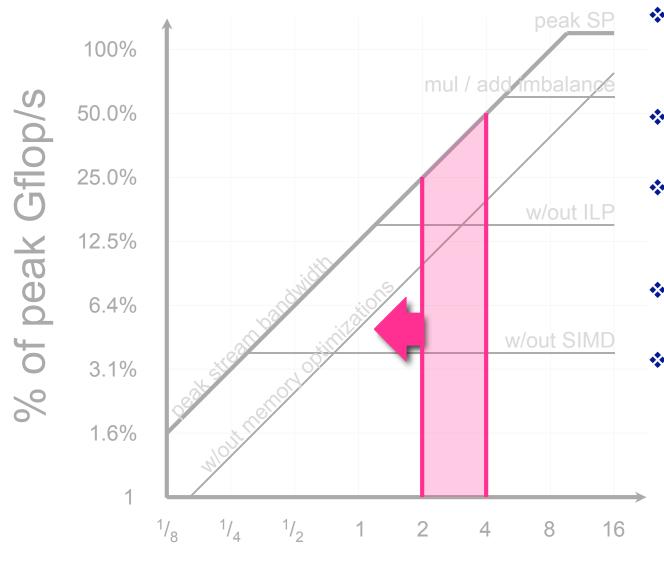




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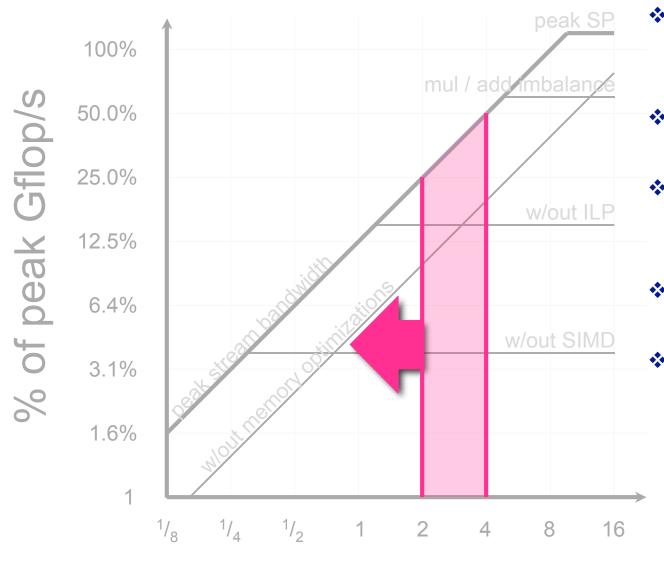




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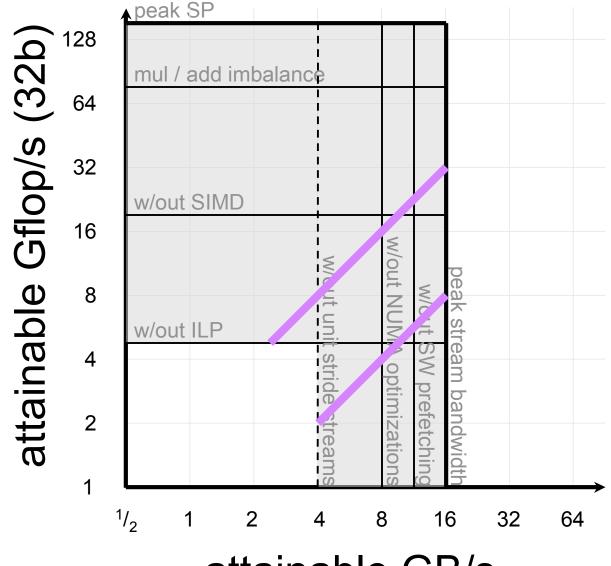


- ILP is decreasing (shorter pipelines, multithreading)
  - SIMD is becoming wider
- Bandwidth isn't keeping up with #cores
- Application flop:byte is decreasing
  - Cache/Local Store management is becoming critical



# **Alternate View (Barcelona)**





- Same formalism can be plotted on different axis
- Difficult to plot Al



## No Overlap



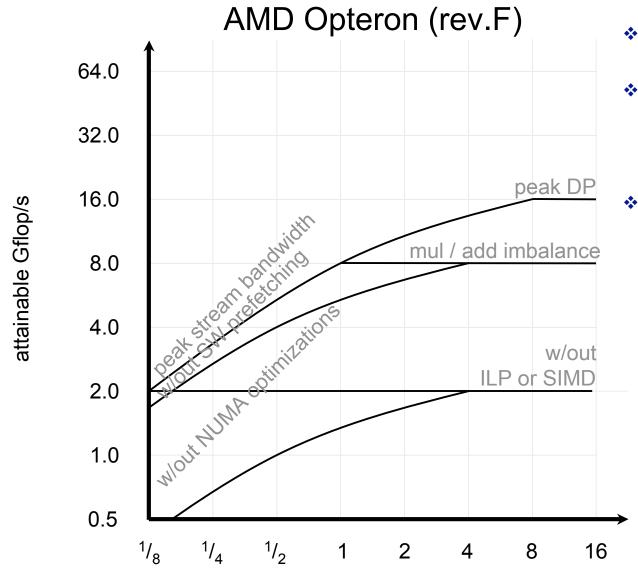
- What if computation or communication isn't totally overlapped
- At ridgepoint 50% of the time is spent in each, so performance is cut in half
- In effect, the curves are smoothed
- Common for bulk synchonous MPI communication, atypical for DRAM access on modern architectures



### Roofline model for Opteron



(non-overlapped)



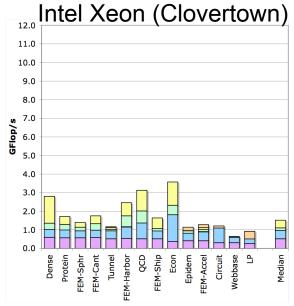
- Not typical of multithread/core architectures
- Not typical of architectures with ooo or HW prefetchers
- More common in network accesses

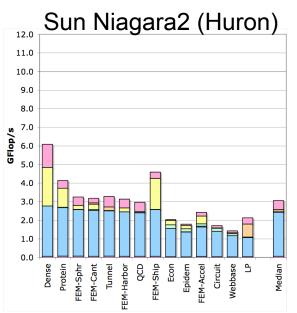


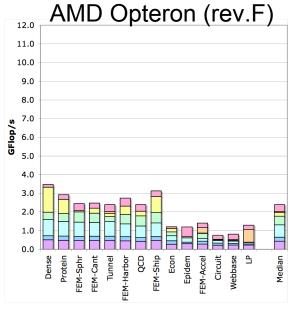
#### **Auto-tuned Performance**

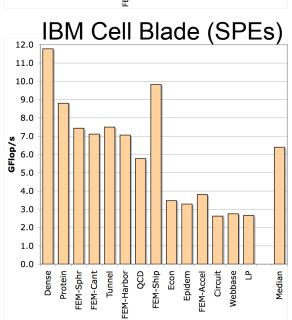


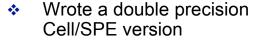
(+Cell/SPE version)



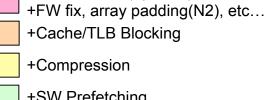




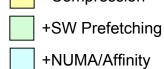


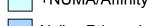


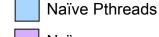
- DMA, local store blocked, NUMA aware, etc...
- Only 2x1 and larger BCOO
- Only the SpMV-proper routine changed
- About 12x faster (median) than using the PPEs alone.



+More DIMMs(opteron),







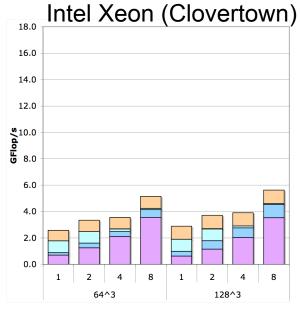


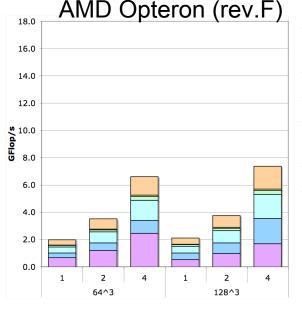


#### **Auto-tuned Performance**



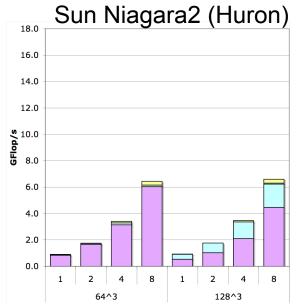
(Local Store Implementation)

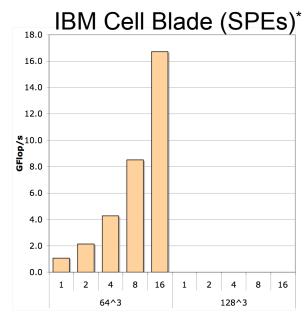






- VL, unrolling, reordering fixed
- No NUMA
- Exploits DMA and double buffering to load vectors
- Straight to SIMD intrinsics.
- Despite the relative performance, Cell's DP implementation severely impairs performance









### Where do GPUs fit in?



	Expressed at compile time	Discovered at run time
Instruction Level Parallelism	VLIW	superscalar, SMT, etc
Data Level Parallelism	SIMD, Vector	G80

 GPUs discover data level parallelism from thread level parallelism at runtime