# Astrophysical applications of the MAESTRO code

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**Abstract.** Convective velocities in a white dwarf leading up to a Type Ia supernova or in the accreted layers on a neutron star preceding an X-ray burst are very subsonic. Under these conditions, sound waves can be neglected, but capturing compressibility effects due to nuclear reactions and background stratification is critical to accurately model the flow. We have developed a new algorithm, MAESTRO, based on a low Mach number formulation that exploits the separation of scales between the fluid velocity and the speed of sound. Here, we provide a brief overview of MAESTRO and present the initial astrophysical applications of the algorithm.

## 1. Introduction

In convectively driven flows, the phenomena of interest evolve on time scales associated with the fluid velocity, which can be orders of magnitude larger than those associated with acoustic wave propagation. Traditional compressible flow solvers typically cannot effectively model these flows. Low Mach number models exploit this separation of scales to derive approximations that allow for large time steps based on fluid velocity rather than sound speed. For large-scale astrophysical applications, a low Mach number model must incorporate a general equation of state [1], must allow for finite-amplitude density and temperature perturbations, and must be able to evolve the base state of the star. We have developed a new low Mach number hydrodynamics code, MAESTRO [2, 3, 4], that accurately and efficiently models these types of astrophysical flows.

In a low Mach number approximation, the pressure is decomposed into a thermodynamic pressure, $p_0$, and a dynamic pressure, $\pi$, such that $\pi/p_0 \sim O(M^2)$, where $M$ is the Mach number of the flow. We define a base state density, $\rho_0$, by requiring the thermodynamic pressure be in hydrostatic equilibrium, $\partial p_0/\partial r = -\rho_0 g$, with gravitational acceleration, $g$. The total pressure is replaced by $p_0$ everywhere except the momentum equation; this decoupling results in sound waves being filtered from the system. This procedure results in the following system of equations:

$$\frac{\partial(\rho X_k)}{\partial t} = -\nabla\cdot(\mathbf{U}\rho X_k) + \rho\dot{\omega}_k \ , \tag{1}$$

$$\frac{\partial(\rho h)}{\partial t} = -\nabla\cdot(\mathbf{U}\rho h) + \frac{Dp_0}{Dt} + \rho H_{\rm nuc} + \rho H_{\rm ext} + \nabla\cdot\kappa\nabla T \ , \tag{2}$$

$$\frac{\partial \mathbf{U}}{\partial t} = -\mathbf{U}\cdot\nabla\mathbf{U} - \frac{1}{\rho}\nabla\pi - \frac{(\rho-\rho_0)}{\rho} g\mathbf{e}_r \ , \tag{3}$$

 

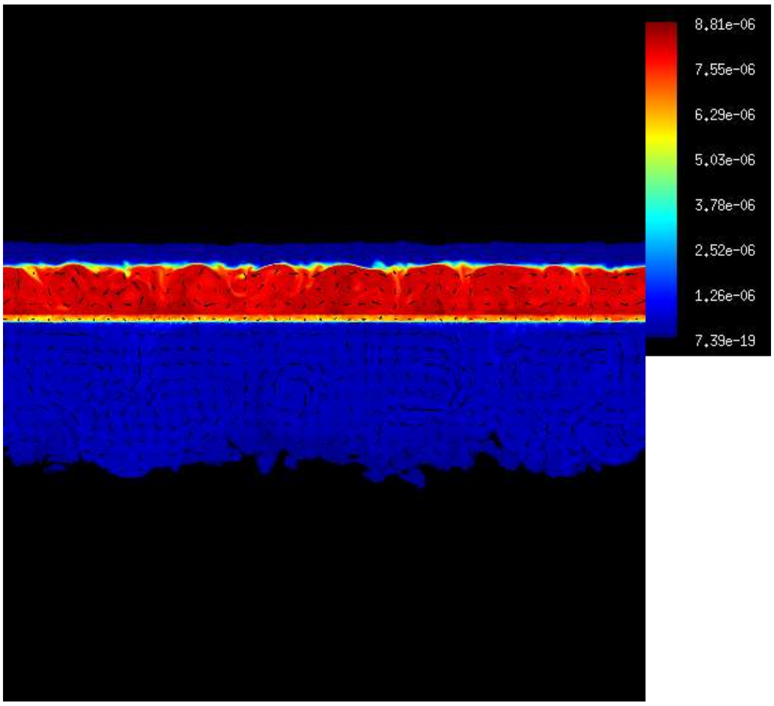


**Figure 1.** Carbon mass fraction over-plotted with velocity vectors after 0.022 s of evolution for the XRB convection problem. (Note we have zoomed in on the convective region—the total vertical extent of the image is 850 cm.)

$$\nabla \cdot (\beta_0 \mathbf{U}) \quad = \quad \beta_0 \left( S - \frac{1}{\overline{\Gamma}_1 p_0} \frac{\partial p_0}{\partial t} \right) \quad . \tag{4}$$

Here, $X_k$ are the mass fractions of the nuclear species, with production rates, $\dot{\omega}_k$. The total mass density, $\rho$, is defined as the sum of the partial densities, $(\rho X_k)$. The specific enthalpy, $h$, has heating sources due to reactions, $H_{\rm nuc}$, (optional) external sources, $H_{\rm ext}$, and thermal diffusion, where $T$ is the temperature and $\kappa$ is the thermal conductivity.

The velocity field, $\mathbf{U}$, satisfies a divergence constraint (4) derived from the equation of state. The presence of the densitylike quantity $\beta_0$ multiplying $\mathbf{U}$ inside the divergence incorporates the compressibility effects due to the background stratification, similar to the effect of $\rho_0$ in the divergence constraint in the anelastic approximation. The source term, $S$, includes the compressibility effects due to heat release and compositional mixing. Finally, $\overline{\Gamma}_1$ is the average of $\Gamma_1 = \partial \log p / \partial \log \rho |_s$ over a layer. Full details of the derivation, the form of $\beta_0$ and $S$, and the solution of this system of equations are given in [4].

As discussed in [3, 4], for the method to accurately represent the physical system, the pressure that follows from the equation of state must remain close to the base state pressure, $p_0$, as the system evolves. To accommodate this requirement, we evolve the base state in response to the large scale heating and convection on the grid. Extensive comparisons to compressible hydrodynamics methods were shown in [2, 3, 4] and to anelastic hydrodynamics in [2], all showing excellent agreement. Furthermore, we have demonstrated that the algorithm scales well up to 1000 processors [5].

## 2. X-ray bursts

The standard picture of an x-ray burst (XRB) is the thermonuclear explosion of the accreted H/He layer on the surface of a neutron star (see [6] for a review). As fuel accretes, the temperature and density at the base of the layer increase until conditions are right for a thermonuclear runaway. It is believed that the runaway begins in a small region and propagates subsonically through the fuel layer [7]. Most theoretical work on the subject has been done in one

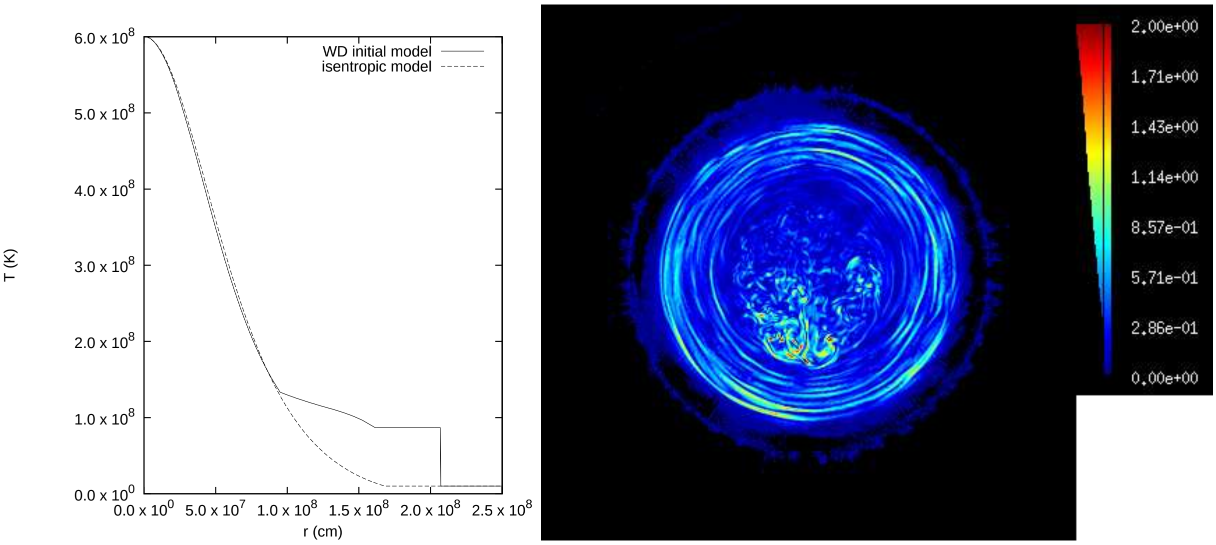


**Figure 2.** Temperature of the initial model compared to an isentropic model (left) and vorticity (X-Z slice) after 814 s of evolution (right) for the convecting WD problem. (Note that the scale for the vorticity has been reduced from the peak value of 3.27 to enhance detail.)

dimension (see [8, 9] for example). However, these models assume the fuel is burned uniformly over the surface of the star which is unlikely if accretion is not spherically symmetric [10]. Models using the shallow-water approximation [11] have shown the importance of the Coriolis force in localizing the burning front. Previous two-dimensional low Mach number calculations presented in [12] showed the development of convection in the fuel layer. While both MAESTRO and the method used in [12] are based on low Mach number approaches, nontrivial differences between the two have been described in [4]. Our goal for the study of XRBs is to extend the calculations of [12] using our low Mach number method to higher resolution, larger domains, and three dimensions. We will also explore the effect of rotation on localization of the burning.

XRBs are well described by plane-parallel geometries, so the algorithm we use is largely that described in [4], with minor changes. The only significant addition is thermal diffusion, which contributes a term in the enthalpy equation (2) and in $S$. The treatment of thermal diffusion follows the procedure described in [13] for small-scale astrophysical flames.

To model the XRB we began with a $1.4 M_\odot$, 10 km neutron star with a thin layer ($\sim 5$ m) of $^4$He on the surface. The composition below the accreted layer is set to pure $^{56}$Fe. This model is taken as the initial base state for the low Mach number algorithm. We model the upper 10–15 m of the neutron star including its surface; this differs from the model of [12], which did not include the surface. Nuclear burning occurs via 3-$\alpha$ reactions as given in [14] and drives convection throughout the He layer. The simulation we present here had a domain size of 640 cm by 1280 cm on a $512 \times 1024$ grid. An initial relative density perturbation of $1.0 \times 10^{-6}$ was centered just above the helium interface to break the symmetry.

Figure 1 shows the $^{12}$C mass fraction over-plotted with velocity vectors after 22 ms of evolution. The sharp interface at the bottom of the red–orange region is the base of the accreted helium layer where 3-$\alpha$ reactions occur most rapidly and drive convection throughout the overlying layers. Additionally there is a large region of convective overshoot below the helium layer, the extent of which seems to have some dependence on the resolution of the model. The formation and persistence of this overshoot region are currently being investigated.

## 3. White dwarf convection

A Type Ia supernova (SN Ia) is the thermonuclear explosion of a white dwarf that has accreted from a companion to the point where it approaches the Chandrasekhar limit (see [15] for a review). At this point, the temperature is hot enough in the center for carbon fusion reactions to begin, driving convection throughout the star. This convection can last for centuries, increasing the overall temperature of the white dwarf [16]. Eventually, the temperature is high enough that the reactions cannot be quenched by the convective motions of rising plumes. At this point, a burning front is said to have ignited, and it will consume the available carbon/oxygen fuel in seconds. Models of exploding white dwarfs typically begin by seeding one or many hot spots at or near the center of the star and following the subsequent evolution (see for example [17, 18, 19, 20]). These calculations demonstrate that an understanding of the convection leading up to ignition is critical, as this is what determines the number, size, and distribution of the hot spots that seed the burning front. There have been a few investigations of this convection [21, 22, 23], but a lot of work remains.

Extending MAESTRO to handle flows with a radial base state requires a procedure for mapping the one-dimensional base state onto the three-dimensional Cartesian grid, and the inverse operation. At present, we use a base state with finer zoning than the Cartesian cells. The gravitational acceleration in this case is computed based on the base state mass distribution. This variation in $g$ leads to a different evolution equation for the base state velocity than that for plane-parallel simulations. As discussed in [4], the temperature, $T$, can be derived through the EOS with either $h$ or $p_0$ as an input in addition to $\rho$ and $X_k$. For spherical flows, we've found that using $T = T(p_0, \rho, X_k)$ reduces the numerical noise associated with representing a spherical star on a Cartesian grid. This remains a topic of active research, and full details of the spherical algorithm will appear in a subsequent paper. The remainder of the algorithm and input physics follows the description in [4].

To model convection, we begin with a Chandrasekhar mass white dwarf initial model, with a central temperature of $6 \times 10^8$ K and a central density of $2.6 \times 10^9$ g cm$^{-3}$. The composition is approximately 0.3 $^{12}$C and 0.7 $^{16}$O by mass, with some heavier nuclei near the center. This model is based on those explored in [22]. The initial model is taken to be the initial base state and mapped onto the Cartesian grid. For the simulation presented here, we used a domain $5 \times 10^8$ cm on a side, with the full star centered on a $384^3$ grid. An initial velocity field consisting of a number of random modes of amplitude $10^5$ cm/s was seeded at the center of the star. Thermonuclear reactions ($^{12}\mathrm{C} + {}^{12}\mathrm{C} \rightarrow {}^{24}\mathrm{Mg} + \gamma$) begin in the center and drive convection throughout the star.

Figure 2 shows the temperature profile of the initial model compared to a purely isentropic model. The departure of our initial model from the isentropic model at $10^8$ cm shows that only the inner part of the star is convectively unstable. The vorticity plot of the right shows this behavior after 13.5 minutes of evolution, with the inner region of the star showing convective behavior. The large descending plume hints at some overall structure to the convective flow, perhaps the onset of a dipole field like that seen in the calculations of [22, 23]. This behavior is still under investigation.

## 4. Conclusions

We have shown some preliminary results of applying MAESTRO to the study of XRBs and SNe Ia. We will continue to explore the robustness of the results presented here to algorithmic improvements and resolution. These initial results demonstrate that the new algorithm is able to model convective behavior in astrophysical environments. Future work will include the development of an adaptive version of the algorithm, which we will initially apply to the XRB problem to better resolve the base of the burning layer. We will also add rotation, for both the XRB and SNe Ia problems, and turbulent flame models.

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