A.2.3

Numerical Calculation of Complex Shock Reflections in Gases

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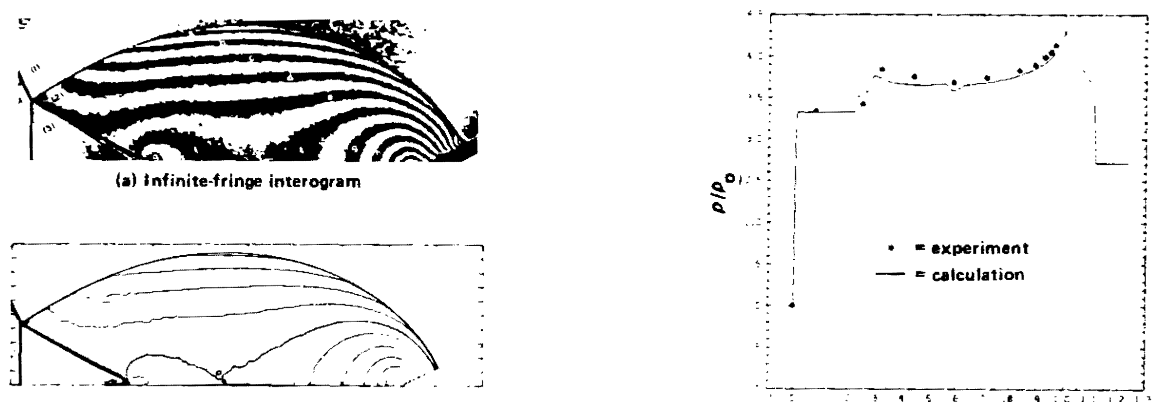
We present here computational results using second order Godunov methods for time-dependent Eulerian gas dynamics with a general (convex) equation of state. The algorithm used in most of the calculations is described in detail in [3]. An unsplit scheme was used in conjunction with local adaptive mesh refinement for the results in Figure 3; see [1] and the references cited there for the construction of this version of the scheme.

For the well-known problem of planar shock wave diffraction by a wedge, a direct comparison of computational results and the experimental record has been completed for several different cases, [5]. Reproduced in Figure 1 are the results for a shock wave Mach number $M_s = 2.03$ and wedge angle $\theta = 27°$. The experimantal flowfield for this case of single Mach reflection is in equilibrium and viscous effects are localized at the wedge corner and the contact surface-boundary layer interaction. The infinite-fringe interferogram is shown in Fig. 1a, the numerical results with the same isopycnics (i.e., constant density lines) in Fig. 1b, and the wall density values are compared in Fig. 1c. It is seen that excellent agreement is obtained in all respects.



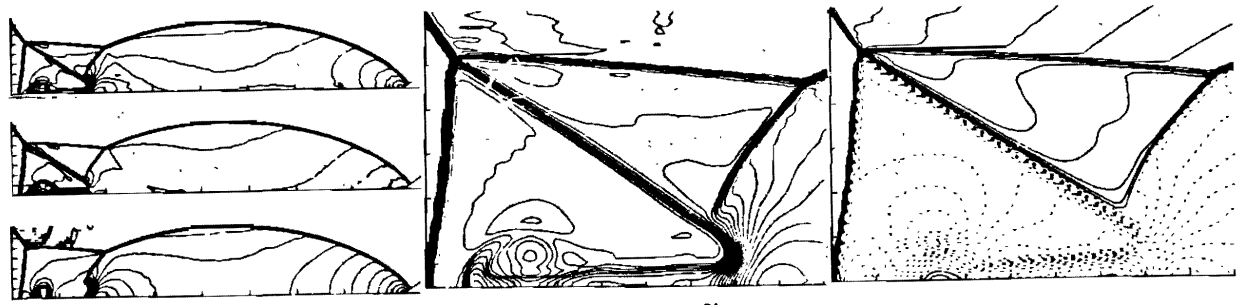
(a) Infinite-fringe interogram

(b) Computed isopycnics

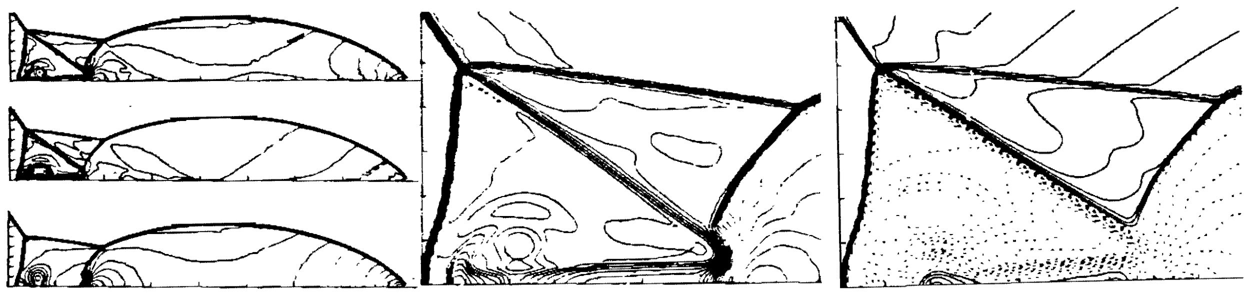
(c) Comparison, ρ/ρ_o vs. x/L

Figure 1. Comparison for planar shock wave diffraction, $M_s = 2.03$, $\theta = 27°$ in air ($\gamma = 1.4$). Here, ρ = density, ρ_o = ambient density, x = distance along wedge surface, L = distance between wedge corner and Mach stem along wedge surface.

A bifurcation study for planar shock wave diffraction by a wedge is presented in Figure 2. All calculations are for a polytropic gas with $M_s = 8.0$ and $\theta = 35°$ The five sets of results are for (a) $\gamma = 1.40$, (b) $\gamma = 1.35$, (c) $\gamma = 1.30$, (d) $\gamma = 1.25$, and (e) $\gamma = 1.20$. We summarize a few of the phenomena here, see [2] for a fuller discussion of this and related flowfields. As the parameter γ decreases, the following transition phenomena occur in the double Mach stem flowfield: (1) the leading Mach stem "toes out" and then forms a new triple point ((c), T_1); for those values of γ, the flow is supersonic in a coordinate system moving with the self-similar solution, and the slip line from that triple point is swept up in the highly rotational flow brought about by the curvature of the shock below T_1. (2) the expansion and compression waves in the supersonic region between the main slip surface and the reflected shock ((d), R) become stronger, until the cross-flow becomes supersonic, causing the compression wave to form into a shock ((d), S_R), (3) the flow inside the jet along the wall becomes transonic, and a backwards facing shock forms at the wall ((e), S_w).

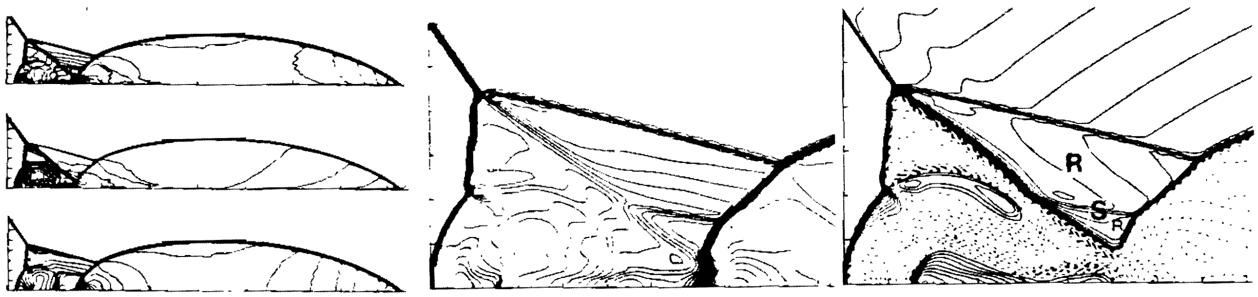


(a) γ = 1.40; (ρ, e, p) whole flowfield; (ρ, $\tilde{M}$) Mach stem region

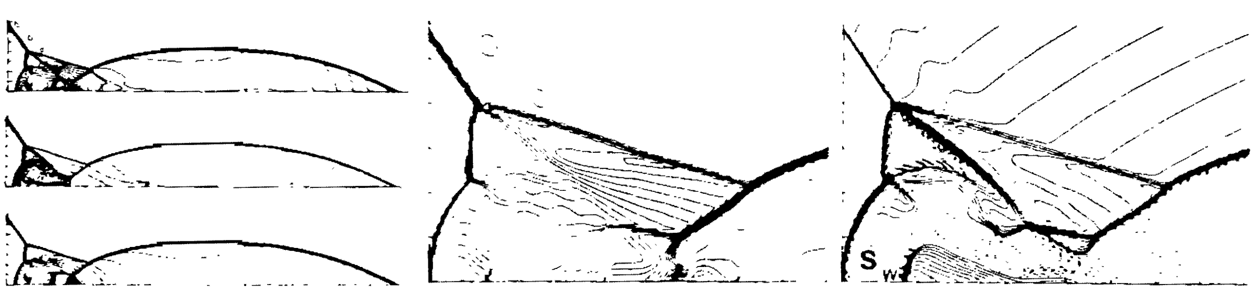


(b) γ = 1.35; (ρ, e, p) whole flowfield; (ρ, $\tilde{M}$) Mach stem region

(c) γ = 1.30; (ρ, e, p) whole flowfield; (ρ, $\tilde{M}$) Mach stem region



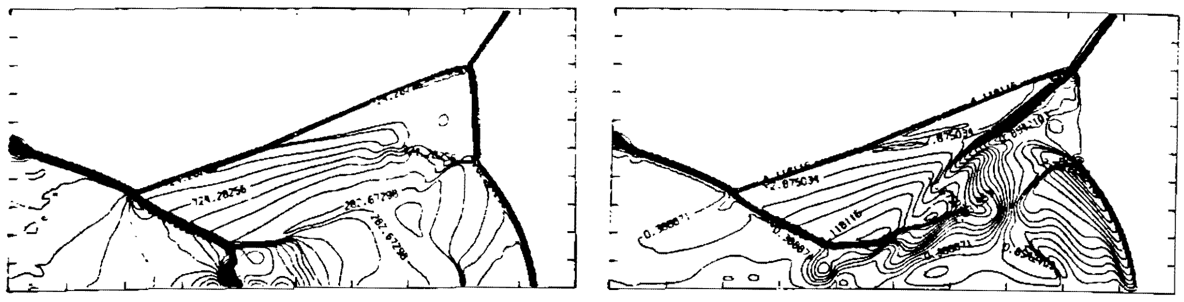
(d) γ = 1.25; (ρ, e,p) whole flowfield; (ρ, $\tilde{M}$) Mach stem region



(e) γ = 1.20; (ρ, e, p) whole flowfield; (ρ, $\tilde{M}$) Mach stem region

Figure 2. Bifurcation study for planar shock wave diffraction, $M_s = 8.0$, $\theta = 35^\circ$. Here, ρ = density, e = internal energy, p = pressure, $\tilde{M}$ = self-similar Mach number, where $\tilde{M}^2 = (\tilde{u}^2+\tilde{v}^2)/C^2$ and $\tilde{u} = u - \xi$, $\tilde{v} = v - \eta$, $(\xi, \eta) = (x/t, y/t)$ with the origin of coordinates at the wedge corner. In the plots of $\tilde{M}$, solid (dashed) lines correspond to $\tilde{M} > 1$ ($\tilde{M} < 1$).

Although it would be possible to continue the series of calculations shown in figure 2 to lower values of γ, the problem is well-suited as an application of the mesh refinement algorithm proposed in [1] for use with the second-order Godunov schemes. Results for the case $M_s = 8.0$, $\theta = 35°$, $\gamma = 1.15$ are shown in Figure 3. For this case, the rarefaction fan emanating from the main contact surface impinges on the reflected shock, and creates a differential in the oblique jump conditions across this wave. As the figure indicates, this wave develops a new (but relatively weak) triple point; the overall configuration may be considered a "triple Mach stem".



(a) Pressure (b) Vertical velocity

Figure 3. Planar shock wave diffraction, $M_s = 8.0, \theta = 35^{\circ}, \gamma = 1.15$. The double Mach stem region is shown and it corresponds to the locally refined (fourfold relative to the coarse mesh) region; the grid shown is 208 x 104 points.

The flowfield resulting from the detonation of an 8 lb. sphere of chemical explosive (PBX-9404) 51.7 cm. above the reflecting surface is presented in Figure 4. Our calculation is hydrodynamic with the initial data provided by a similarity analysis of the detonation process (see [4]). The calculation is performed in cylindrical coordinates using a rectangular moving grid. the entire grid is 100 cm. x 20 cm. and the fine grid is 8 cm. x 4 cm. The fine grid contains the point at which the incident shock wave intersects the surface. The coarse grid to fine grid ratio was 10 for this calculation. Transition from regular to double Mach reflection takes place in the calculation when the incident shock reaches about 57 cm. Since this is infinitesimal phenomenon, we expect a somewhat earlier transition for the grid strategies in [1],[2]. The similarities between the double Mach stem flowfield and the flowfields in Figure 2 may be noted.

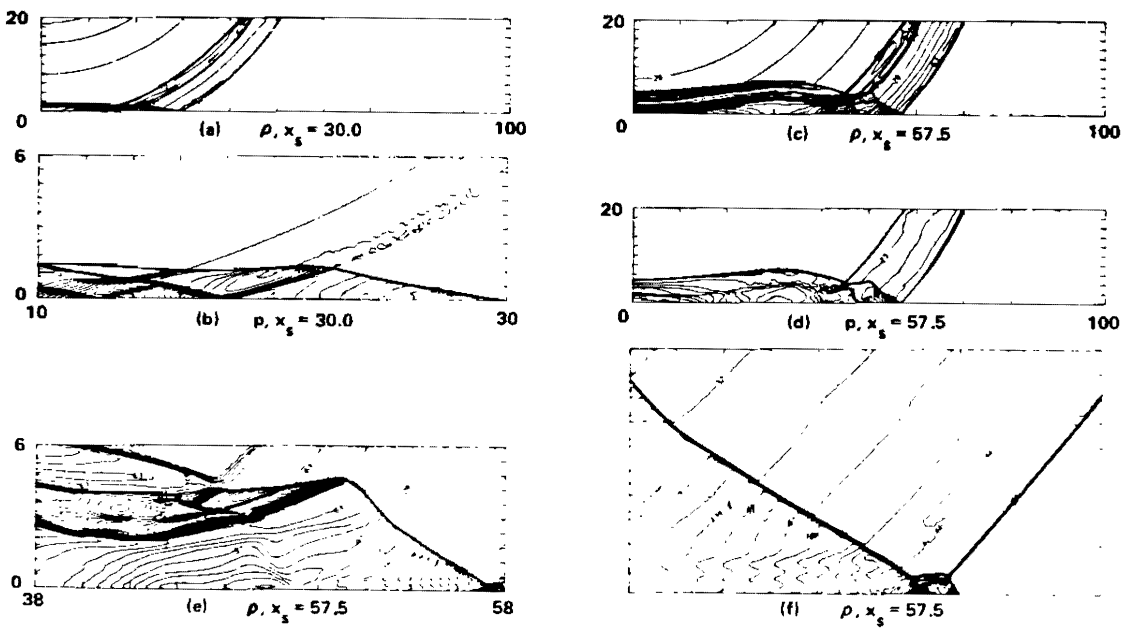


Figure 4. Continued on next page.

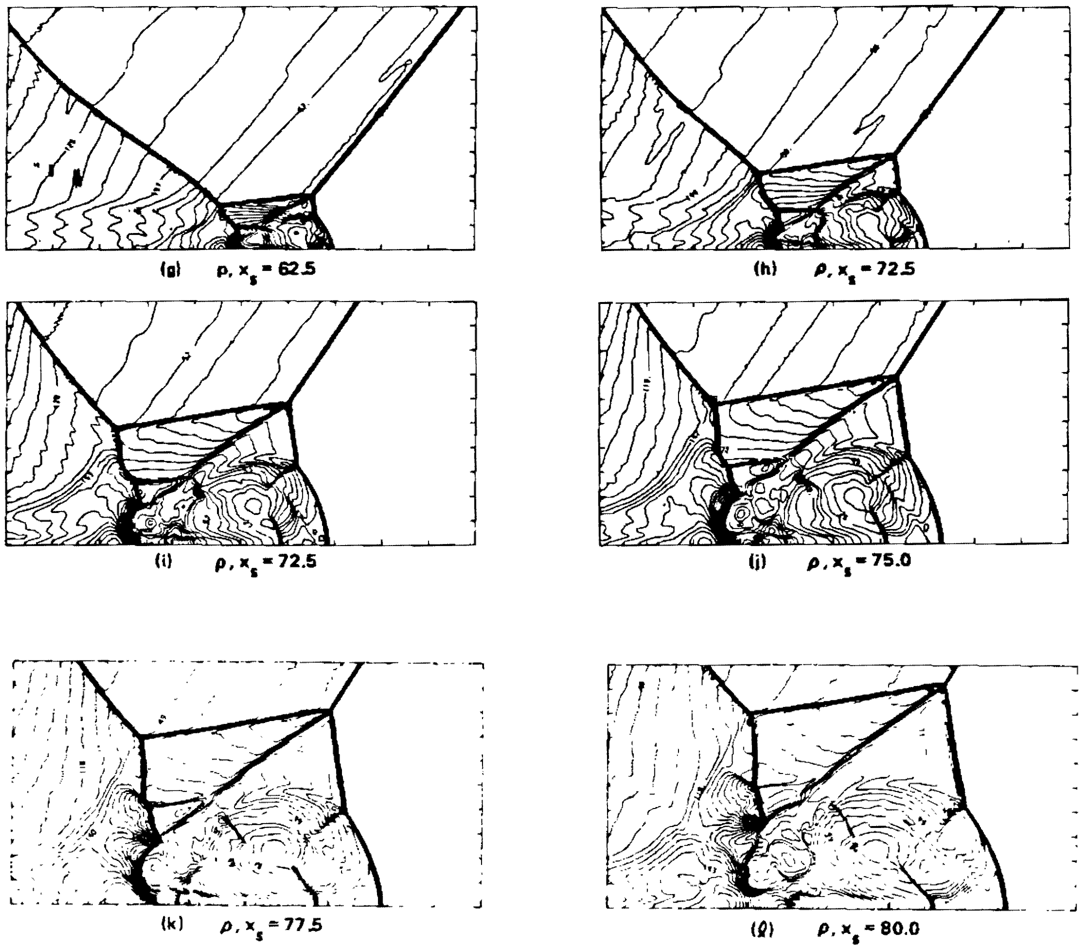


Figure 4. Blast wave flowfield. Plots (a), (c), (d) represent the entire grid; plots (b), (e) are 20 cm x 6 cm blowups; plots (f) - (ℓ) represent the entire 8 cm x 4 cm find grid. Here, ρ = density, p = pressure, x_s = location of intersection of incident shock and the surface. The grid contains 617 x 214 zones with 266 x 140 zones in the fine grid.

The interaction of a planar shock with a five zone thick layer of heated gas located five zones above a reflecting surface is shown in Figure 5. The gas is assumed polytropic with $\gamma = 1.4$. The solution is symmetric with respect to the centerline of the layer until waves from the lower boundary of the layer reach the surface. The interaction of the resulting reflected waves with the symmetric rollup leads to instabilities and the production of counterrotating vortices at later times.

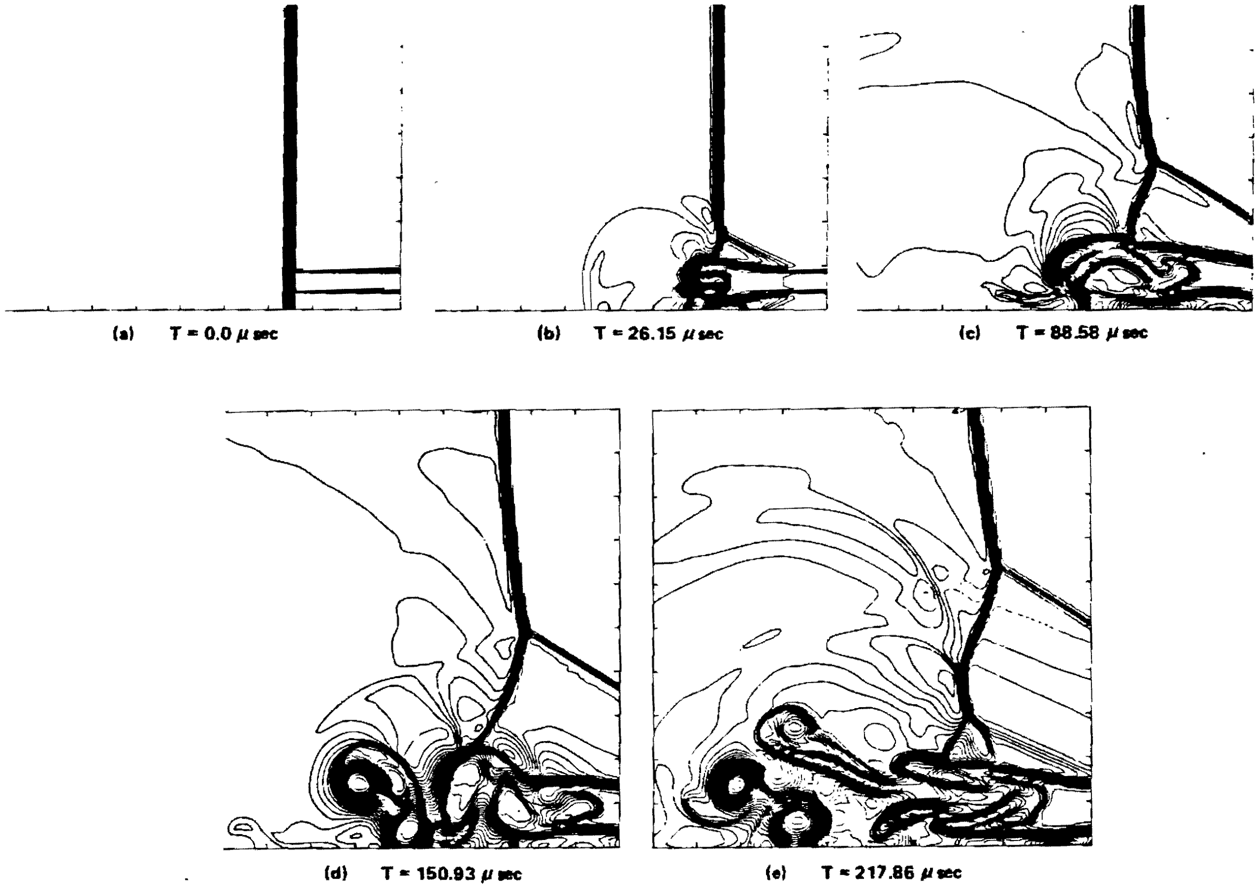


Figure 5. Planar shock wave interaction with a layer of heated gas. The shock faces towards the right, T = 273°K in the ambient gas on the right above and below the heated layer, for which T = 2124°K. Pressure is atmospheric everywhere on the right and Δp = 30 bars across the shock ($P_L/P_R \sim 3$). $\Delta x = \Delta y \lessapprox 0.05$ cm everywhere. Denisty contours are shown in the vicinity of the incident shock wave.

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