



BERKELEY LAB

LAWRENCE BERKELEY NATIONAL LABORATORY



U.S. DEPARTMENT OF
ENERGY

HPGMG Benchmark

Samuel Williams

Lawrence Berkeley National Laboratory

SWWilliams@lbl.gov

Goal was to create a benchmark that is...

- Application- and Mathematically-relevant
- Scale-free and independent of parallelization
- Precise in its definition
- Sufficiently simple that undergraduates can optimize it
- Exercises aspects of system architecture not addressed by HPL or HPCG

HPGMG-FV

- Solves Variable Coefficient Poisson... $Lu = \nabla \cdot \beta \nabla u = f$
- Discretized with the 4th order Finite Volume Method
- Solved on a Cubical Cartesian grid with Dirichlet Boundary Conditions
- Uses the asymptotically exact Full Multigrid (FMG)
- Fully specified stencils and smoothers
- public git repo: <https://bitbucket.org/hpgmg/hpgmg>

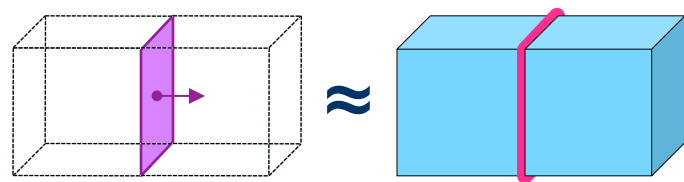
Finite Volume Method

- In FVM, the average value of an operator over a cell's volume ($\langle \nabla \cdot \beta \nabla u \rangle$) is computed as an integral over the cell's surface ($\langle \beta \nabla u \cdot \mathbf{N} \rangle$).
- For 3D structured grids, each h^3 cell has 6 faces and we must calculate this flux term on each of the 6 faces on every cell in the entire domain....

$$Lu = \langle \nabla \cdot \beta \nabla u \rangle = \left[\text{cube with left face shaded and arrow pointing left} \right] + \left[\text{cube with right face shaded and arrow pointing right} \right] + \left[\text{cube with front face shaded and arrow pointing down} \right] + \left[\text{cube with back face shaded and arrow pointing up} \right] + \left[\text{cube with bottom face shaded and arrow pointing down} \right] + \left[\text{cube with top face shaded and arrow pointing up} \right]$$

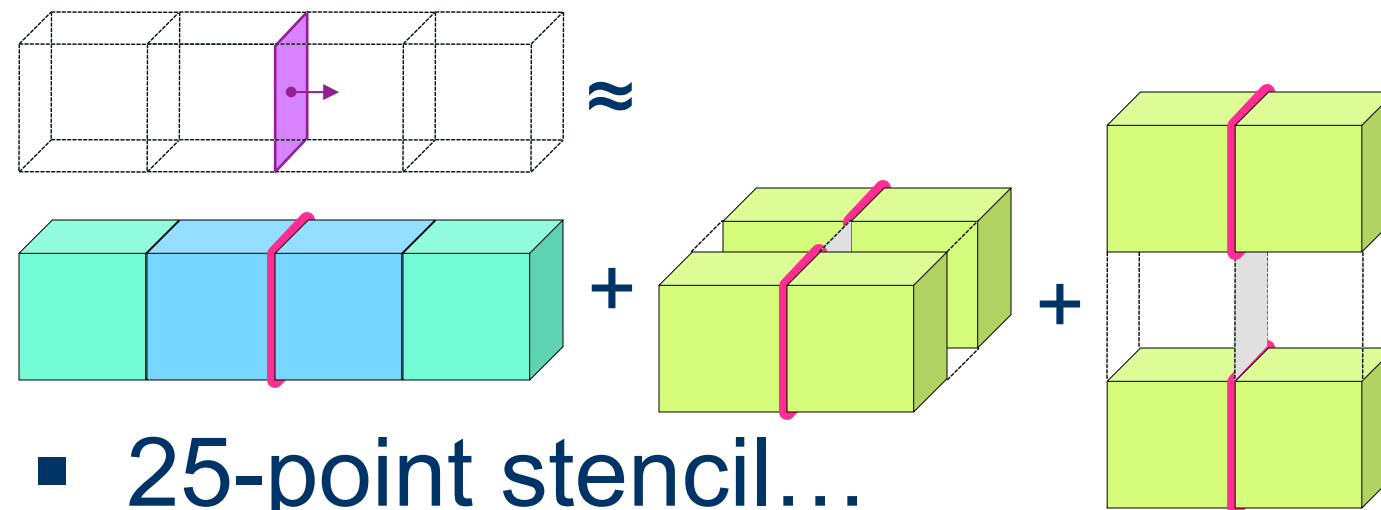
4th Order Operator $\langle \nabla \cdot \beta \nabla u \rangle$

- In 2nd order, we can approximate each of these flux terms as a 2-point weighted stencil...



- 6 such terms form a 7-point variable-coefficient stencil.
 - Low arithmetic intensity
 - 6 MPI messages/smooth

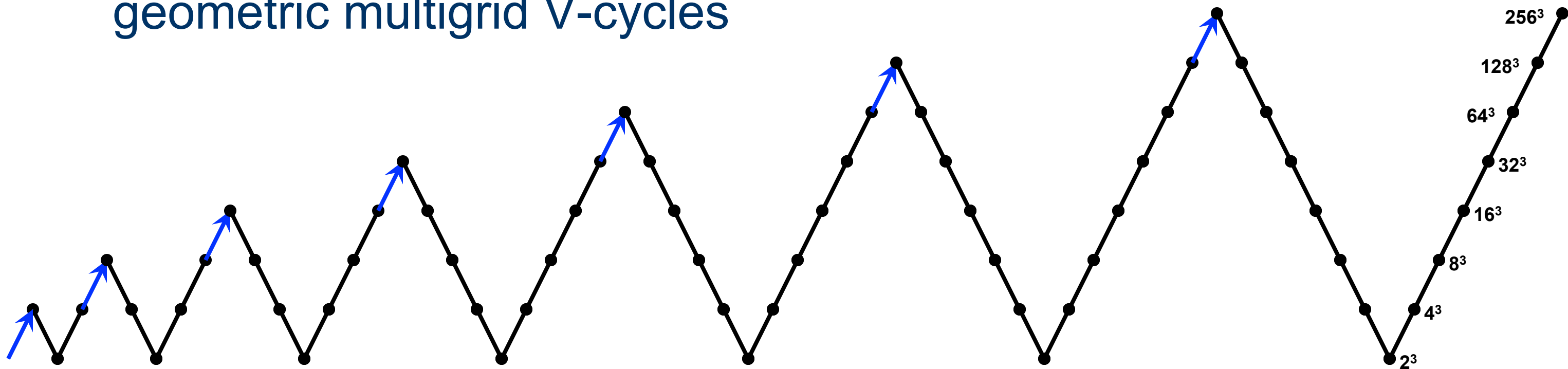
- For 4th order, additional terms are required...



- 25-point stencil...
 - 9 (unique) 4-point stencils
 - ~no extra DRAM data movement
 - 4x the floating-point operations
 - 3x the MPI messages/smooth
 - 2x the MPI message size

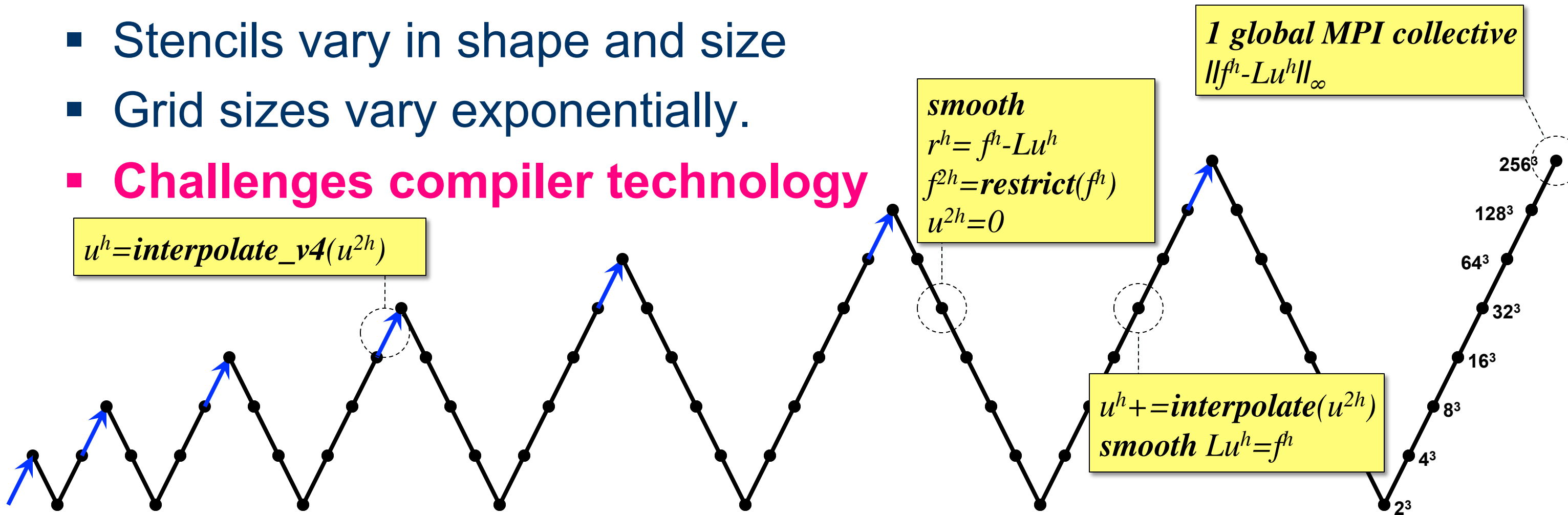
HPGMG uses Full Multigrid (FMG)

- FMG is a single pass, direct solver that provides a solution to the discretization error (4th order)
- The FMG multigrid F-Cycle is a series of progressively deeper geometric multigrid V-cycles



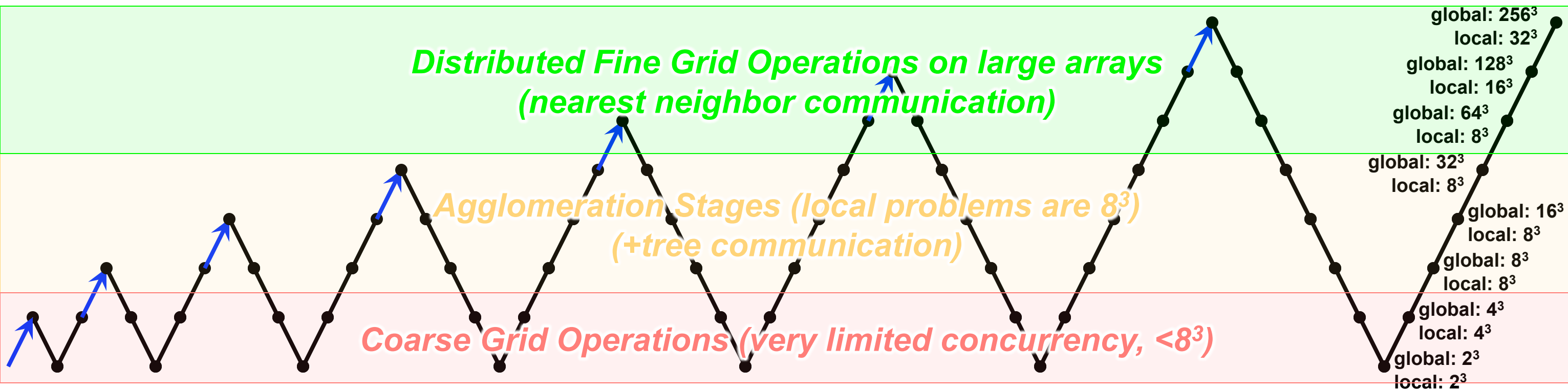
HPGMG Uses Many Different Stencils

- Several dozen stencil sweeps per step
- Stencils vary in shape and size
- Grid sizes vary exponentially.
- **Challenges compiler technology**



HPGMG Has Multiple Communication Patterns

- Work is redistributed onto fewer cores (agglomeration)
- Coarse grid solves can occur on a single core of a single node
- Coarse grid solution is propagated to every thread in the system



November 2016 Ranking

HPGMG Rank	System Site	System Name	10 ⁹ DOF/s	MPI	OMP	Acc	DOF per Process	Top500 Rank	Notes
1	ALCF	Mira	500	49152	64	0	36M	9	BGQ
2	HLRS	Hazel Hen	495	15408	12	0	192M	14	
3	OLCF	Titan	440	16384	4	1	32M	3	K20x GPU
4	KAUST	Shaheen II	326	12288	16	0	144M	15	
5	NERSC	Edison	296	10648	12	0	128M	60	
6	CSCS	Piz Daint	153	4096	8	1	32M	8	K20x GPU
7	Tohoku University	SX-ACE	73.8	4096	1	0	128M	-	vector
8	LRZ	SuperMUC	72.5	4096	8	0	54M	36	
9	NREL	Peregrine	10.0	1024	12	0	16M	-	
10	NREL	Peregrine	5.29	512	12	0	16M	-	
11	HLRS	SX-ACE	3.24	256	1	0	32M	-	vector
12	NERSC	Babbage	0.762	256	45	0	8M	-	KNC

HPGMG BoF

- Wednesday, November 16 @ 5:15pm in 250-F
- Agenda
 - Discuss latest HPGMG rankings
 - HPGMG-FV on Intel's Knights Landing manycore processor
 - HPGMG-FV on NVIDIA GPUs including Pascal
 - HPGMG's communication patterns and vectorization on the SX-ACE
 - Open discussion