Performance Tuning with the **Roofline Model on GPUs and CPUs**

2:30pm 2:35pm 3:15pm 4:00pm 4:30pm 5:00pm 5:30pm 5:55pm

Welcome Introduction to Roofline **Roofline on GPUs (basics)**

break **Roofline on GPUs (advanced) Roofline on CPUs Application Use Cases** closing remarks / Q&A

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Introductions

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Introduction to the Roofine Model

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- Imagine running a mix of benchmarks on a new system (e.g. GPU/CPU)...
 - GFLOP/s alone may not be particularly insightful
 - speedup relative to the previous system may seem random
- We need a quantitative model that defines <u>Good Performance</u>





- Good Performance is tied to "Efficient" execution
- Two fundamental requirements
 - 1. Must operate the CPU/GPU in the throughput-limited regime not sensitive to Amdahl effects, D2H/H2D transfers, launch overheads, etc...
 - 2. Must attain high utilization of the CPU/GPU's compute and/or **bandwidth** capabilities



Roofline Model

- **Roofline Model** is a throughput-oriented performance model
- applies to x86, ARM, POWER CPUs, GPUs, Google TPUs¹, FPGAs, etc...
- Helps quantify **Good Performance**



https://crd.lbl.gov/departments/computer-science/PAR/research/roofline



- Modern architectures can be complex
- Don't model / simulate full architecture
- Make assumptions on performance and usage...



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- Make assumptions on performance and usage...
 - Peak GFLOP/s on data in L1
 - Load-balanced SPMD code
 - Sufficient cache bandwidth/capacity





- Modern architectures can be complex
- Don't model / simulate full architecture
- Make assumptions on performance and usage...
 - Peak GFLOP/s on data in L1
 - Load-balanced SPMD code
 - Sufficient cache bandwidth/capacity
 - Basis for DRAM Roofline Model





- Any given loop nest will perform:
 - o Computation (e.g. FLOPs)
 - Communication (e.g. moving data to/from DRAM)
- With perfect overlap of communication and computation...



 \circ $\,$ Run time is determined by whichever is greater $\,$

Time = max { #FLOPs / Peak GFLOP/s #Bytes / Peak GB/s



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Time
#FLOPs1 / Peak GFLOP/s#Bytes / #FLOPs / Peak GB/s



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#FLOPs
Time= min { Peak GFLOP/s
(#FLOPs / #Bytes) * Peak GB/s



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 - Computation (e.g. FLOPs)
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AI (Arithmetic Intensity) = FLOPs / Bytes (as presented to DRAM)





Arithmetic Intensity

- Measure of data locality (data reuse)
- Ratio of <u>Total Flops</u> performed to <u>Total Bytes</u> moved
- For the DRAM Roofline...
 - Total Bytes to/from DRAM
 - \circ $\,$ Includes all cache and prefetcher effects $\,$
 - Can be very different from total loads/stores (bytes requested)
 - Equal to ratio of sustained GFLOP/s to sustained GB/s (time cancels)



(DRAM) Roofline Model

GFLOP/s = min { AI * Peak GB/s

AI (Arithmetic Intensity) = FLOPs / Bytes (moved to/from DRAM)

- Plot Roofline bound using Arithmetic Intensity as the x-axis
- Log-log scale makes it easy to doodle, extrapolate performance along Moore's Law, etc...





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'Machine Balance'



(DRAM) Roofline Model

GFLOP/s = min { Peak GFLOP/s AI * Peak GB/s

AI (Arithmetic Intensity) = FLOPs / Bytes (moved to/from DRAM)

 Roofline tessellates this 2D view of performance into 5 regions...



Transition @ AI == Peak GFLOP/s / Peak GB/s == 'Machine Balance'



- Typical machine balance is 5-10
 FLOPs per byte...
 - o 40-80 FLOPs per double to exploit compute capability
 - o Artifact of technology and money
 - o Unlikely to improve

Consider STREAM Triad...

#pragma omp parallel for for(i=0;i<N;i++){ Z[i] = X[i] + alpha*Y[i]; }

- 2 FLOPs per iteration
- Transfer 24 bytes per iteration (read X[i], Y[i], write Z[i])
- AI = 0.083 FLOPs per byte == Memory bound





Conversely, 7-point constant coefficient stencil...



<pre>#pragma omp parallel for</pre>
<pre>for(k=1;k<dim+1;k++){< pre=""></dim+1;k++){<></pre>
<pre>for(j=1;j<dim+1;j++){< pre=""></dim+1;j++){<></pre>
<pre>for(i=1;i<dim+1;i++){< pre=""></dim+1;i++){<></pre>
new[k][j][i] = -6.0*old[k][j][i]
+ old[k][j][i-1]
+ old[k][j][i+1]
+ old[k][j-1][i]
+ old[k][j+1][i]
+ old[k-1][j][i]
+ old[k+1][j][i];
}}}



- Conversely, 7-point constant coefficient stencil...
 - o 7 FLOPs
 - o 8 memory references (7 reads, 1 store) per point
 - AI = 7 / (8*8) = 0.11 FLOPs per byte (measured at the L1)





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 - 7 / (8+8) = 0.44 FLOPs per byte (DRAM)

```
#pragma omp parallel for
for(k=1;k<dim+1;k++){</pre>
for(j=1;j<dim+1;j++){</pre>
for(i=1;i<dim+1;i++){</pre>
 new[k][j][i] = -6.0*old[k]
                             + old[k ][j
                                   ][i-1]
                                   ][i+1]
                     + old[k
                              1[i
                     + old[k
                              _][i–1][i
                     + old[k ][i+1][i
                     + old[k-1][i
                                     1ſi
                     + old[k+1][i ][i
                                          1:
}}}
```

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		DR	A





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 - 8 memory references (7 reads, 1 store) per point Ο
 - Ideally, cache will filter all but 1 read and 1 write per point Ο
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== memory bound, but 5x the FLOP rate as TRIAD

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}}}



Peak GFLOP/s GFLOP/s ≤ AI * DRAM GB/s

7-point Stencil



 Think back to our mix of loop nests (benchmarks)...





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- We can sort kernels by their arithmetic intensity...





- Think back to our mix of loop nests (benchmarks)
- We can sort kernels by their arithmetic intensity...
- ... and compare performance relative to machine capabilities





Kernels near the roofline are making good use of computational resources





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 - kernels can have <u>low performance</u> (GFLOP/s), but make good use (%STREAM) of a machine





- Kernels near the roofline are making good use of computational resources
 - kernels can have <u>low performance</u> (GFLOP/s), but make good use (%STREAM) of a machine
 - kernels can have <u>high performance</u> (GFLOP/s), but still make poor use of a machine (%peak)





How can performance ever be below the Roofline?







How can performance be below the Roofline?

- Does one always attain either...
 - Peak DRAM Bandwidth \bigcirc
 - Peak FLOP/s
- Theoretical vs. Empirical
 - Use benchmarked GFLOP/s and GB/s \bigcirc
 - Application FLOPs can be underestimated Ο (how many FLOPs is a divide?)
- Bottlenecks other than DRAM and FLOP/s...
 - Insufficient cache bandwidth + locality Ο
 - Didn't use FMA / Vectors / Tensors / ... \bigcirc
 - Too many non-FP instructions Ο
 - Load imbalance; not SPMD Ο etc...







Below the Roofine? Theoretical vs. Empirical







Machine Characterization

- Theoretical performance (specs) can be highly optimistic...
 - DRAM pin bandwidth vs. sustained Ο
 - TurboMode / Underclocking 0
 - compiler failing on high-AI loops. Ο
- Need empirical performance data
- LBL developed the Empirical Roofline Toolkit (ERT)...
 - Characterize CPU/GPU systems Ο
 - Peak Flop rates Ο
 - Bandwidths for each level of memory Ο
 - **MPI+OpenMP/CUDA == multiple GPUs** 0



GFLOPs / sec




Theoretical vs. Empirical

Theoretical Roofline:

- Pin bandwidth Ο
- FPUs * GHz \bigcirc
- 1 C++ FLOP = 1 ISA FLOPΟ
- Data movement = Compulsory Misses Ο



Theoretical GFLOP/s



Theoretical vs. Empirical (Machine)

Theoretical Roofline:

- o Pin bandwidth
- o FPUs * GHz
- 1 C++ FLOP = 1 ISA FLOP
- Data movement = Compulsory Misses

Empirical Roofline:

- Measured bandwidth
- Measured Peak FLOP/s





Theoretical vs. Empirical (Application FLOPs)

Theoretical Roofline:

- Pin bandwidth \bigcirc
- FPUs * GHz \bigcirc
- 1 C++ FLOP = 1 ISA FLOP \bigcirc
- Data movement = Compulsory Misses Ο
- **Empirical Roofline**:
 - Measured bandwidth \bigcirc
 - Measured Peak FLOP/s \bigcirc
 - 1 C++ FLOP >= 1 ISA FLOP (e.g. divide)Ο





Empirical **GFLOP**/s Al using

empirical FLOPs



Theoretical vs. Empirical (Application Bytes)

Theoretical Roofline:

- Pin bandwidth \bigcirc
- FPUs * GHz \bigcirc
- 1 C++ FLOP = 1 ISA FLOP \bigcirc
- Data movement = Compulsory Misses Ο
- **Empirical Roofline**:
 - Measured bandwidth \bigcirc
 - Measured Peak FLOP/s \bigcirc
 - 1 C++ FLOP >= 1 ISA FLOP (e.g. divide)Ο
 - Measured data movement (cache effects) Ο
 - **True Arithmetic Intensity can be higher** or lower than expected





Empirical

GFLOP/s

Frue AI using empirical FLOPs & empirical Bytes



Below the Roofine? Memory Hierarchy and Cache Bottlenecks







- Processors have multiple levels of memory/cache
 - \circ Registers
 - o L1, L2, L3 cache
 - HBM/HBM (KNL/GPU device memory)
 - DDR (main memory)
 - NVRAM (non-volatile memory)





Processors have different bandwidths for each level





- Processors have different bandwidths for each level
 - o different machine balances for each level





- Processors have different bandwidths for each level
 - different machine balances for each level \bigcirc
- Applications have locality in each level
 - different data movements for each level Ο





- Processors have different bandwidths for each level
 - \circ $\,$ different machine balances for each level
- Applications have locality in each level
 - \circ different data movements for each level
 - o different arithmetic intensity for each level



Arithmetic Intensity

GFLOPs L1 GB GFLOPs L2 GB GFLOPs L3 GB GFLOPs DRAM GB



For each additional level of the memory hierarchy, we can add another term to our model...

 AI_x (Arithmetic Intensity at level "x") = FLOPs / Bytes (moved to/from level "x")



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Al_x (Arithmetic Intensity at level "x") = FLOPs / Bytes (moved to/from level "x")



- Plot equation in a single figure...
 - "Hierarchical Roofline" Model 0





- Plot equation in a single figure...
 - "Hierarchical Roofline" Model Ο
 - Bandwidth ceiling (diagonal line) for each Ο level of memory





- Plot equation in a single figure...
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 - Bandwidth ceiling (diagonal line) for each Ο level of memory
 - Arithmetic Intensity (dot) for each level of Ο memory





- Plot equation in a single figure...
 - "Hierarchical Roofline" Model \bigcirc
 - Bandwidth ceiling (diagonal line) for each Ο level of memory
 - Arithmetic Intensity (dot) for each level of Ο memory
 - performance is ultimately the minimum of these bounds





- Plot equation in a single figure...
 - "Hierarchical Roofline" Model \bigcirc
 - Bandwidth ceiling (diagonal line) for each Ο level of memory
 - Arithmetic Intensity (dot) for each level of Ο memory
 - performance is ultimately the minimum of these bounds
- If L2 bound, we see DRAM dot well below DRAM ceiling





Widely separated Arithmetic Intensities indicate high reuse in the cache



Arithmetic Intensity (FLOP:Byte)



- Widely separated Arithmetic Intensities indicate high reuse in the cache
- Similar Arithmetic Intensities indicate effectively no cache reuse (== streaming)



Arithmetic Intensity (FLOP:Byte)





Below the Roofine? FMA, Vectorization, Tensor Cores







Return of CISC

- Vectors have their limits (finite DLP, register file energy scales with VL, etc...)
- Death of Moore's Law is reinvigorating Complex Instruction Set Computing (CISC)
- Modern CPUs and GPUs are increasingly reliant on special (fused) instructions that perform multiple operations (fuse common instruction sequences)...
 - FMA (Fused Multiply Add): z=a*x+y ...*z*,*x*,*y* are vectors or scalars Ο
 - 4FMA (Quad FMA): z=A*x+z ... A is a FP32 matrix; x,z are vectors Ο
 - WMMA (Tensor Core): Z=AB+C ...A, B are FP16 matrices; Z, C are FP32 Ο
- > If instructions are a mix or scalar (predicated), vector, and matrix operations, performance is now a weighted average of them.





Return of CISC

- Consider NVIDIA Volta GPU...
 ~100 TFLOPs for FP16 Tensor
 - 15 TFLOPS for FP32 FMA
 - \circ $\,$ 7.5 TFLOPs for FP32 Add $\,$
- DL applications mix Tensor, FP16, and FP32
- DL performance may be well below nominal Tensor Core peak





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- DL applications mix Tensor, FP16, and FP32
- DL performance may be well below nominal Tensor Core peak
- The actual mix of instructions introduces an <u>effective ceiling</u> on performance...





Below the Roofline? FPU Starvation

ERKFL

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- Processors have finite instruction fetch/decode/issue bandwidth
- Moreover, the number of FP units dictates the FP issue rate required to hit peak
- Ratio of these two rates is the minimum FP instruction fraction required to hit peak



- Consider...
 - 4-issue superscalar Ο
 - 2 FP data paths Ο
 - >50% of the instructions must be FP to have any chance at peak performance





Peak GFLOP/s ≥50% FP 25% FP (75% int) 12% FP (88% int)

Conversely,

- Keeping 2 FP data paths, Ο
- but downscaling to 2-issue superscalar Ο
- 100% of the instructions must be FP to get peak performance





Conversely,

- Keeping 2 FP data paths,
- \circ but downscaling to 2-issue superscalar
- > 100% of the instructions must be FP to get peak performance





Conversely,

- Keeping 2 FP data paths,
- \circ but downscaling to 2-issue superscalar
- 100% of the instructions must be FP to get peak performance
- Codes that would have been memorybound are now decode/issue-bound.





100% FP

50% FP (50% int)







Recap

Roofline bounds performance as a function of Arithmetic Inte

- Horizontal Lines = Compute Ceilings
- Diagonal Lines = Bandwidth Ceilings
- Bandwidth ceilings are parallel on log-log scale
- Collectively, ceilings define an upper limit on performance
- Loop Arithmetic Intensity (for each level of memory)
 - Total FLOPs / Total Data Movement
 - o Includes <u>all</u> cache effects
 - Measure of a loop's temporal locality
- Plotting loops on the Roofline
 - Each loop has one dot per level of memory
 - x-coordinate = arithmetic intensity at that level
 - o y-coordinate = performance (e.g. GFLOP/s)
 - Proximity to associated ceiling is indicative of a performance bound
 - Position of dots relative to each other is indicative of cache locality





What is Roofline used for?

- Understand performance differences between Architectures, Programming Models, implementations, etc...
 - Why do some Architectures/Implementations move more data than others? Ο
 - Why do some compilers outperform others? Ο
- Predict performance on future machines / architectures
 - Set realistic performance expectations 0
 - Drive for HW/SW Co-Design Ο
- Identify performance bottlenecks & motivate software optimizations
- Determine when we're done optimizing code
 - Assess performance relative to machine capabilities Ο
 - Track progress towards optimality Ο
 - Motivate need for algorithmic changes Ο





Model is just one piece of the puzzle...

Roofline Model defines the basic concepts and equations.

Roofline Model (Theory)







Model is just one piece of the puzzle...

 System Characterization defines the shape of the Roofline (peak bandwidths and FLOP/s)



Roofline Model (Theory)

System Characterization (Benchmarking)





Model is just one piece of the puzzle...

- **Application Characterization** determines...
 - Intensity and Performance of each loop Ο
 - Position of any implicit ceilings Ο







Application **Characterization** (Instrumentation)




Model is just one piece of the puzzle...

Visualization tools combine all data together and provide analytical capability











- Charlene will demonstrate how to apply the Roofline model to NVIDIA GPUs
 - o GPU benchmarking
 - \circ application characterization







- Charlene will demonstrate how to apply the Roofline model to NVIDIA GPUs
 - o GPU benchmarking
 - \circ application characterization
- Sam will extend this by examining advanced GPU topics...
 - o Instruction Roofline Model
 - Using Roofline to analyze DL codes
 - Scaling Trajectories







- Charlene will introduce and demo Intel[®] Advisor
 - Automatically instruments applications (one dot per loop nest/function)
 - ✓ Computes FLOPS and AI for each function (CARM)
 - Integrated Cache Simulator (hierarchical roofline / multiple Al's)
 - ✓ AVX-512 support that incorporates masks
 - Automatically benchmarks target system (calculates ceilings)
 - ✓ Full integration with existing Advisor capabilities





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 Jack will discuss how Roofline is used by the NERSC NESAP teams



System Characterization (Benchmarking)

















Performance Extrapolations





Setting Realistic Expectations...

- Consider 3 kernels (A,B,C)
 - kernels A and B are bound by memory bandwidth
 - kernel C is bound by peak FLOP/s





Setting Realistic Expectations...

- Imagine you want to run on a machine with twice the peak FLOPs...
 - kernel C's performance could double
 - X kernels A and B will be no faster





Setting Realistic Expectations...

- What if that machine also doubled memory bandwidth...
 - kernel A and B's performance could also double





Retrospective





- Too many components contribute to app/loop run time...
 - \circ some are characteristics of the application
 - \circ $\,$ some are characteristics of the machine $\,$
 - some are both (memory access pattern + caches)

#FP operations FLOP/s
Cache data movement Cache GB/s
DRAM data movement DRAM GB/s
PCIe data movement PCIe bandwidth
Depth OMP Overhead
MPI Message Size Network Bandwidth
MPI Send:Wait ratio Network Gap
#MPI Wait's Network Latency



Performance models often conceptualize the system as being dominated by one or more aspects of machine and application...

Computational Complexity #FP operations FLOP/s Cache data movement Cache GB/s DRAM data movement DRAM GB/s PCIe data movement PCIe bandwidth **Computational Depth** OMP Overhead MPI Message Size Network Bandwidth MPI Send: Wait ratio Network Gap #MPI Wait's Network Latency



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> **#FP operations FLOP/s** Cache data movement Cache GB/s DRAM data movement DRAM GB/s PCIe data movement PCIe bandwidth Computational Depth OMP Overhead MPI Message Size Network Bandwidth MPI Send: Wait ratio Network Gap LogP #MPI Wait's Network Latency



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Alexandrov, et al, "LogGP: incorporating long messages into the LogP model one step closer towards a realistic model for parallel computation", SPAA, 1995.





• Architectural innovations \rightarrow Throughput Limited Regime

Roofline **#FP operations FLOP/s** Model Cache data movement Cache GB/s DRAM data movement DRAM GB/s PCIe data movement PCIe bandwidth Computational Depth OMP Overhead MPI Message Size Network Bandwidth MPI Send: Wait ratio Network Gap #MPI Wait's Network Latency

