Auto-tuning Multigrid with PetaBricks

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Algorithmic Choice in Sorting

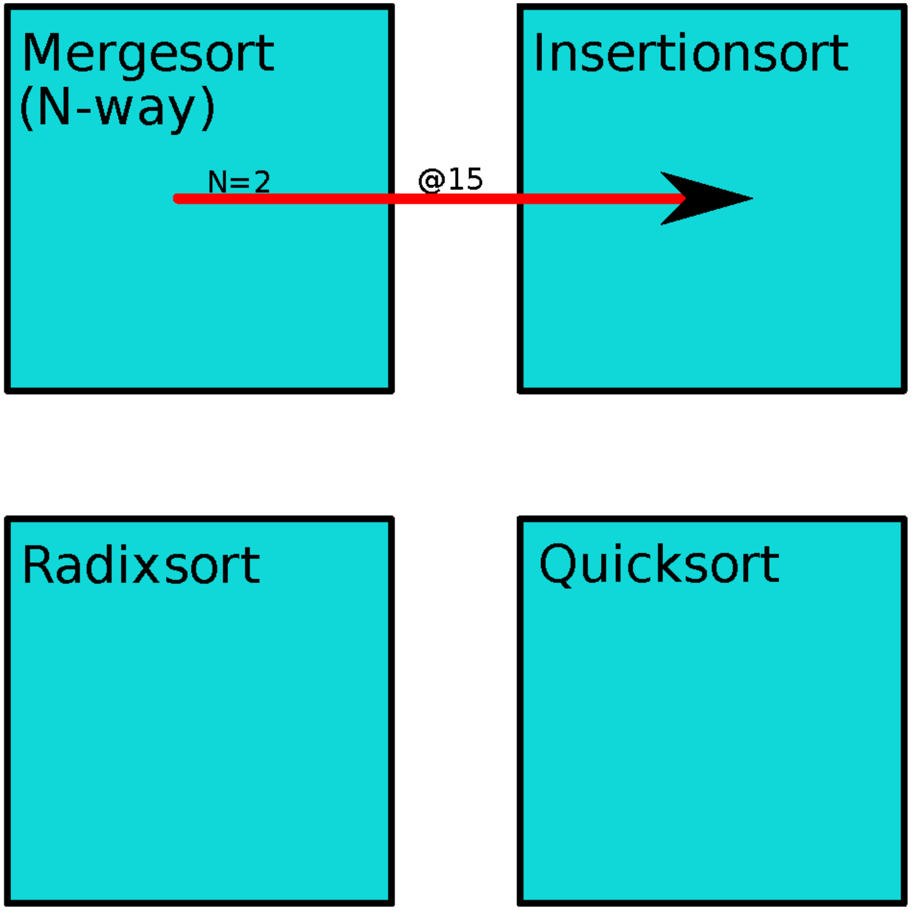
Mergesort (N-way)

Insertionsort

Radixsort

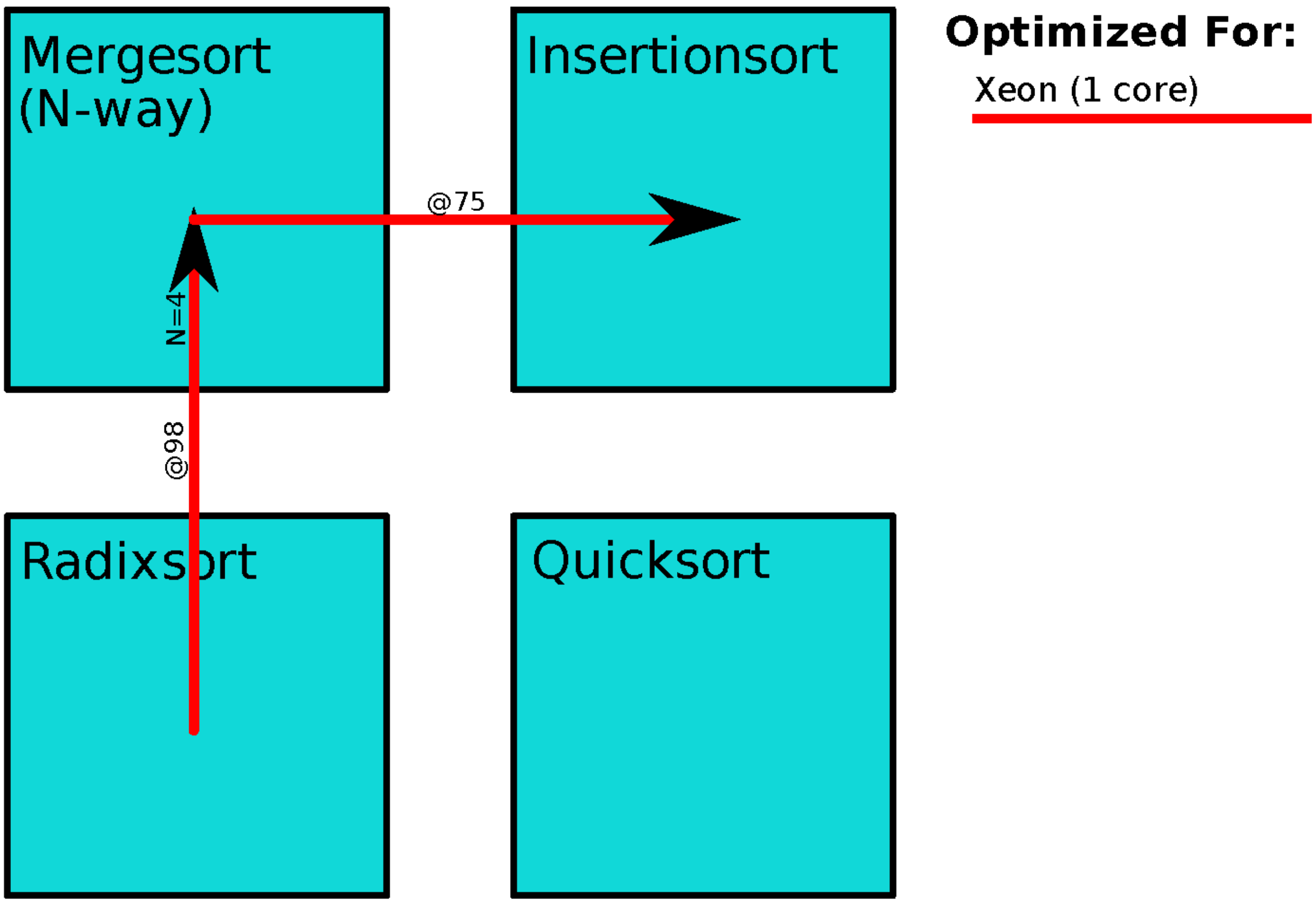
Quicksort

Algorithmic Choice in Sorting

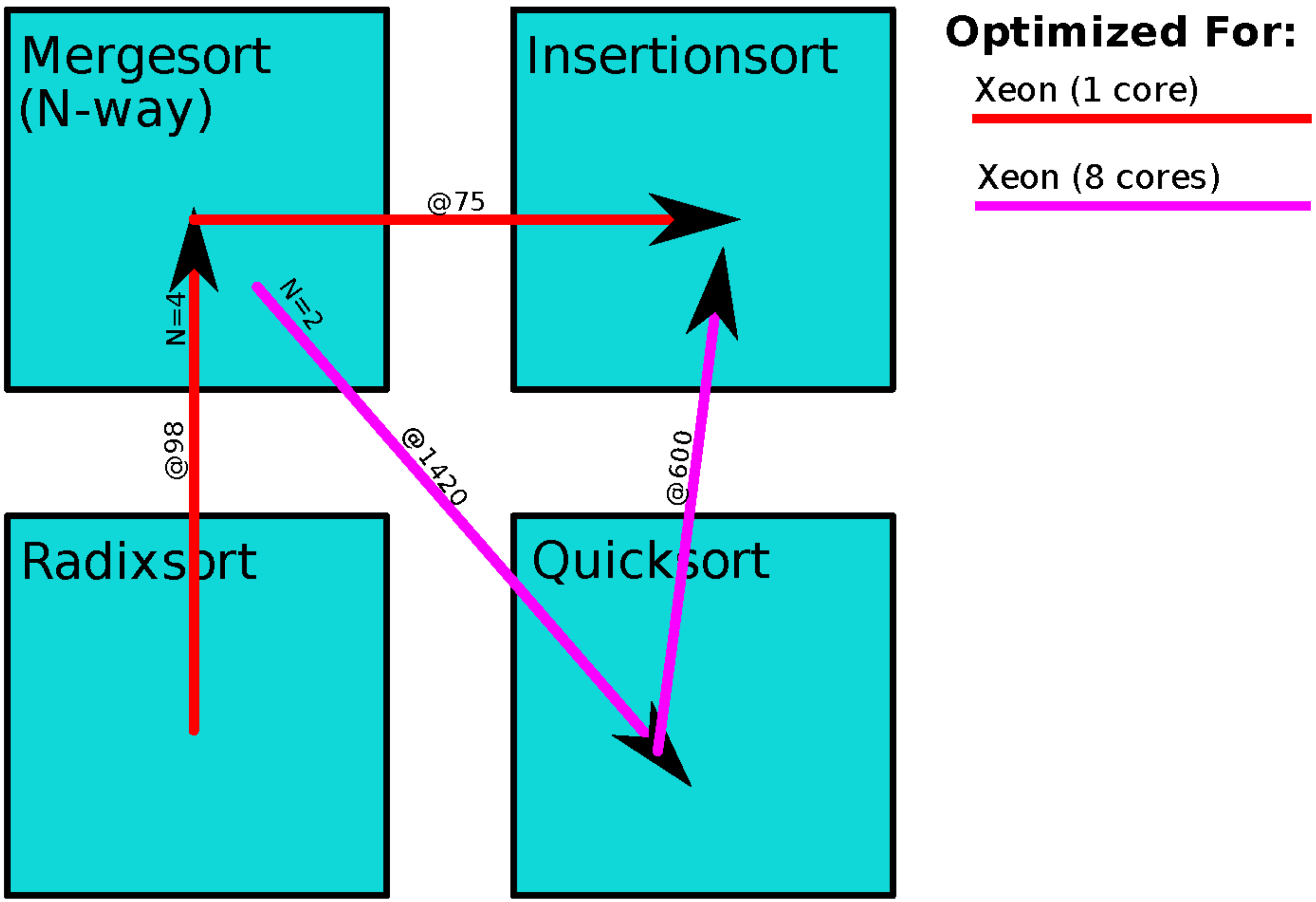


STL Algorithm

Algorithmic Choice in Sorting



Algorithmic Choice in Sorting



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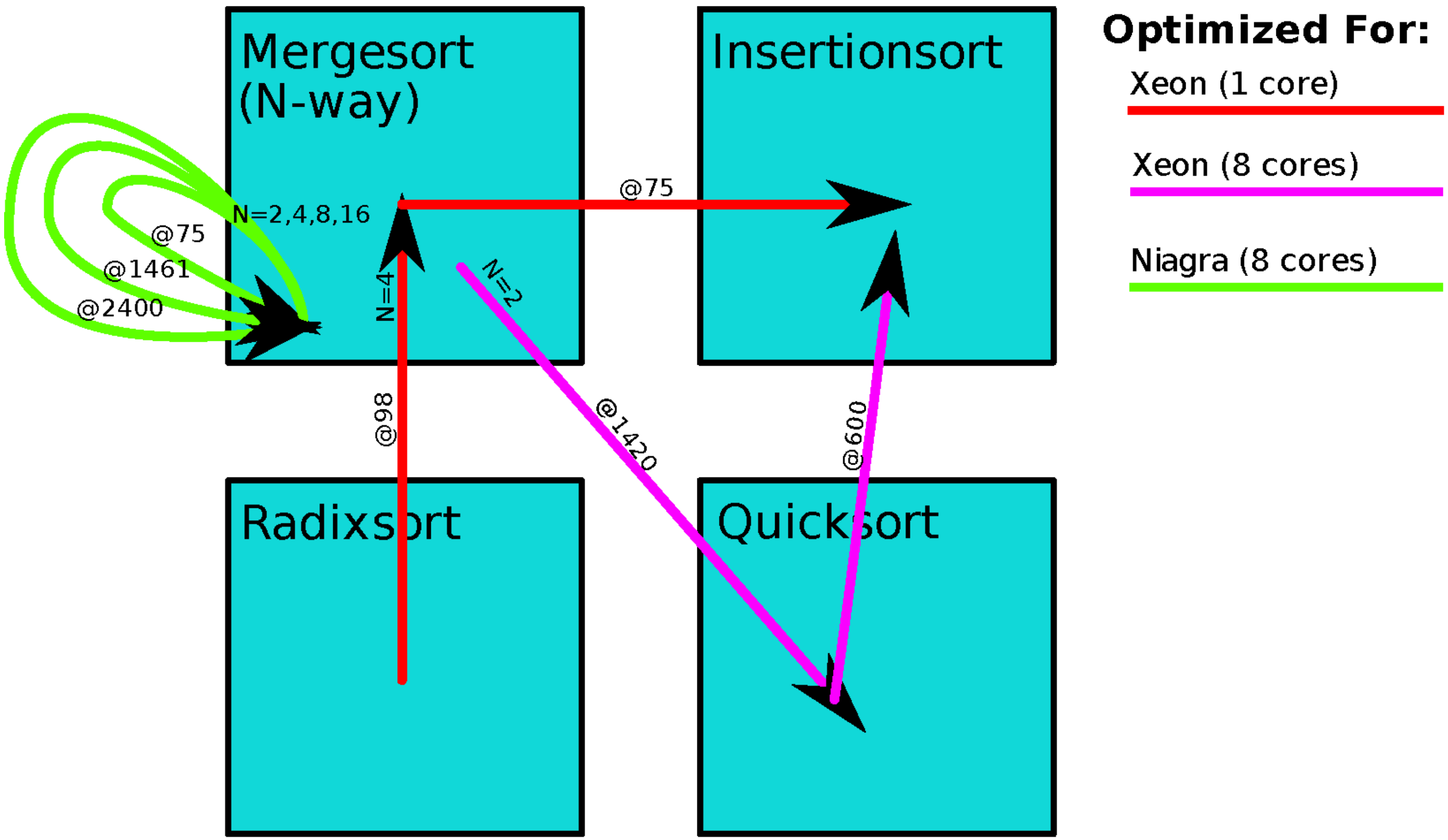


Variable Accuracy Algorithms

- Lots of algorithms where the accuracy of output can be tuned:
  - Iterative algorithms (e.g. solvers, optimization)
  - Signal processing (e.g. images, sound)
  - Approximation algorithms
- Can trade accuracy for speed
- All user wants: Solve to a certain accuracy as fast as possible using whatever algorithms necessary!

The PetaBricks Language

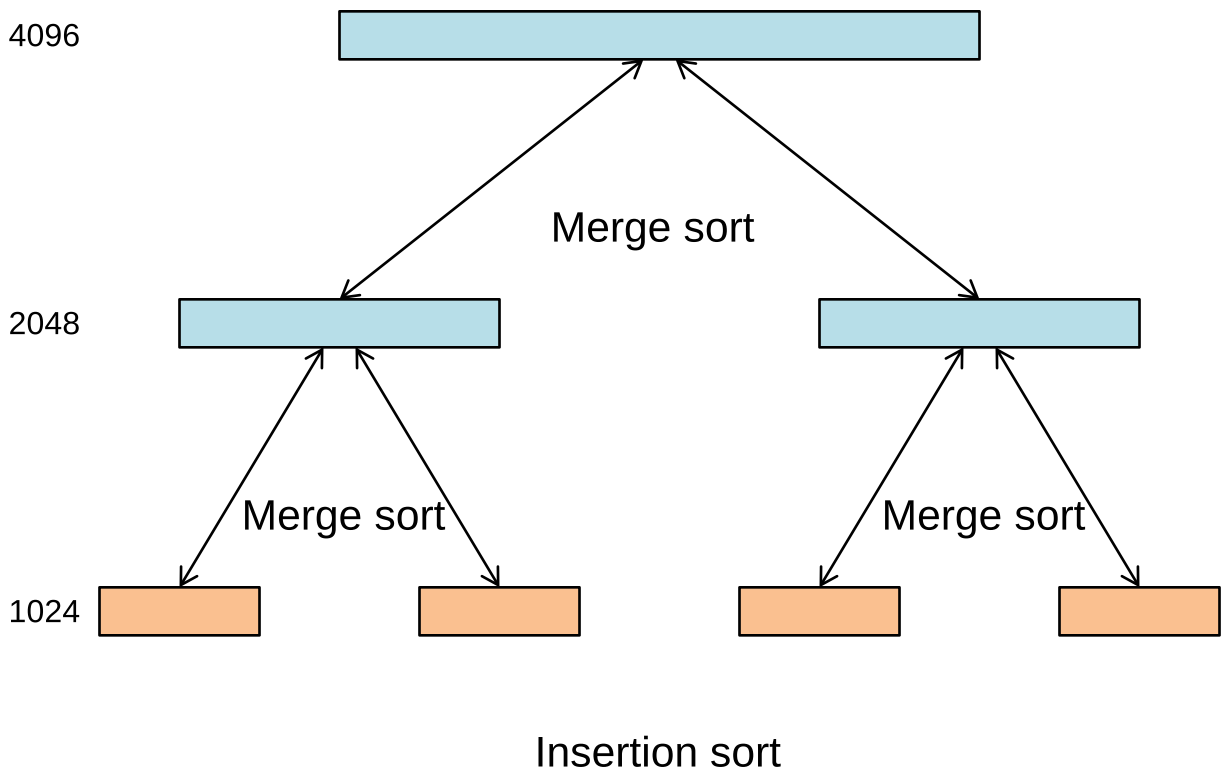
- **General purpose** language and auto-tuner
- Support for algorithmic choices and variable accuracy built into the language
- Specify multiple algorithms and accuracy levels
- Auto-tune parameters (e.g. number of iterations) to produce programs of different accuracy
- Multigrid is a prime target:
  – Iterative linear solver algorithm
  – Lots of choices!

Outline

- Auto-tuning with PetaBricks
- Tuning the Multigrid V-Cycle
- Extension to Auto-tuning Full Multigrid Cycles
- Performance Results

PetaBricks Language Example: Sort

```
transform Sort
from A[n]
to B[n]
{
  // Recursive case, merge sort
  to(B b)
  from(A a) {
    (a1, a2) = Split(a);
    b1 = Sort(a1);
    b2 = Sort(a2);
    b = Merge(b1, b2);
  }

  OR

  // Base case, insertion sort
  to(B b)
  from(A a) {
    b = InsertionSort(a);
  }
}
```



Modeling Costs

Algorithmic Complexity

Compiler Complexity

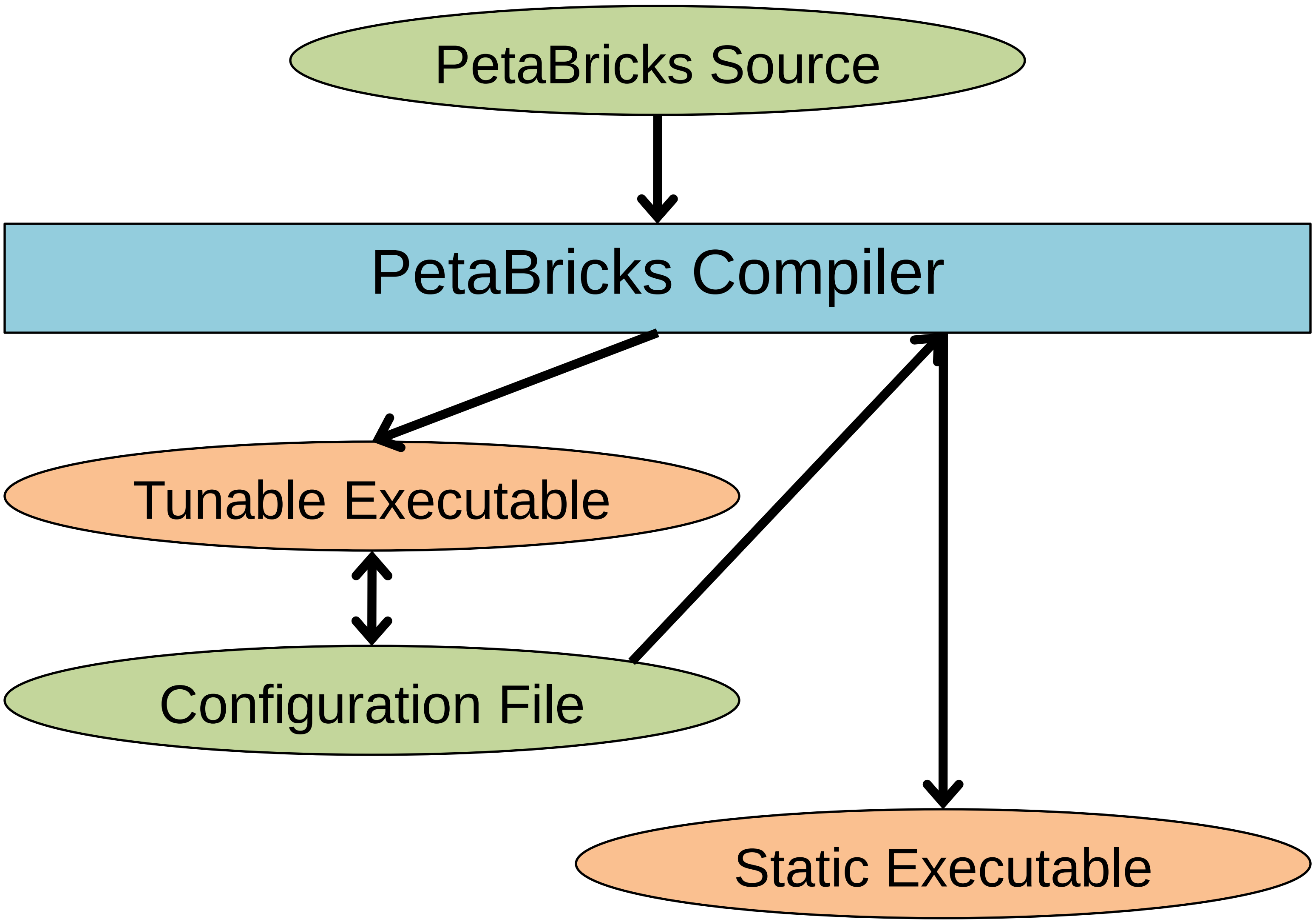
Memory System Complexity

Processor Complexity

All impact performance

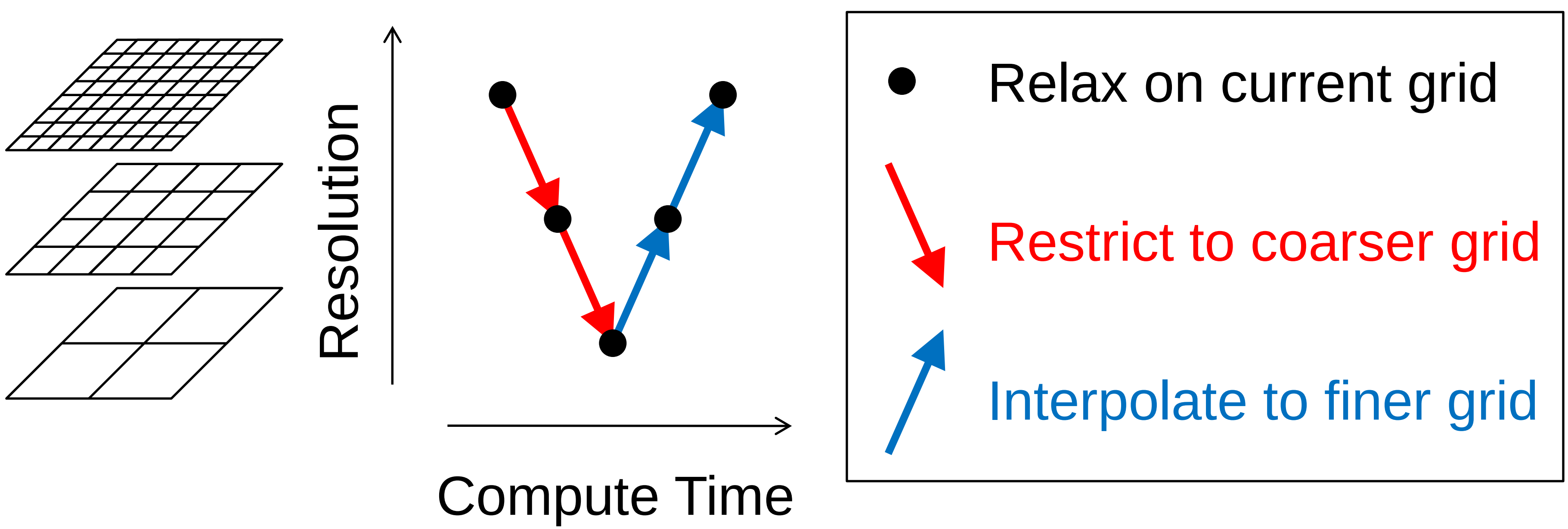
- No simultaneous model for all of these!
- Solution: Use learning!

PetaBricks Work Flow

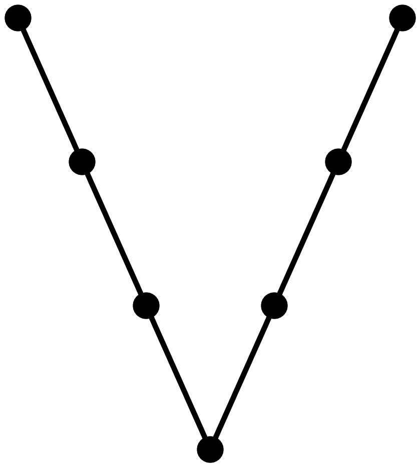


A Very Brief Multigrid Intro

- Used to iteratively solve PDEs over a gridded domain
- **Relaxations** update points using neighboring values (stencil computations)
- **Restrictions** and **Interpolations** compute new grid with coarser or finer discretization

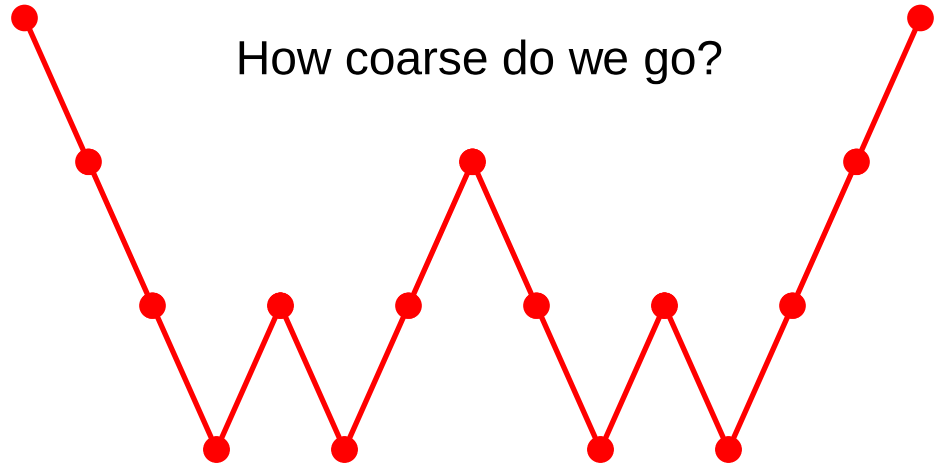


Multigrid Cycles



V-Cycle

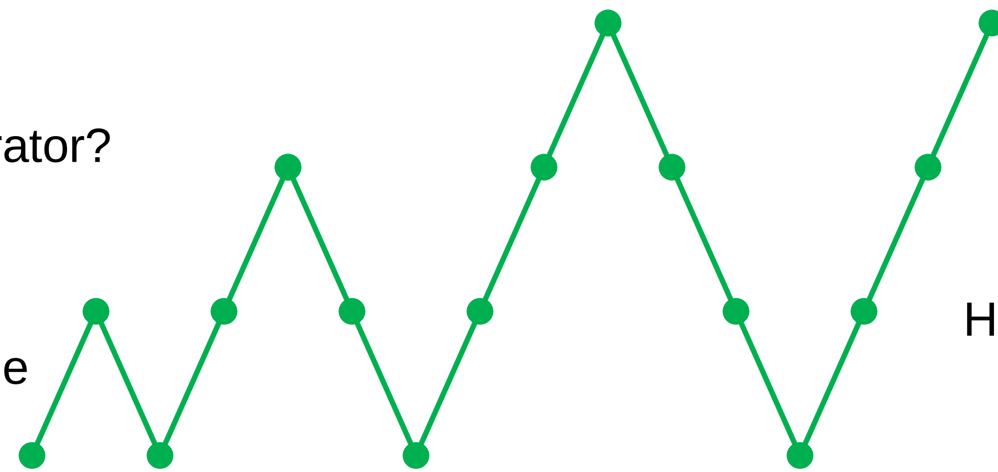
How coarse do we go?



W-Cycle

Relaxation operator?

Full MG V-Cycle

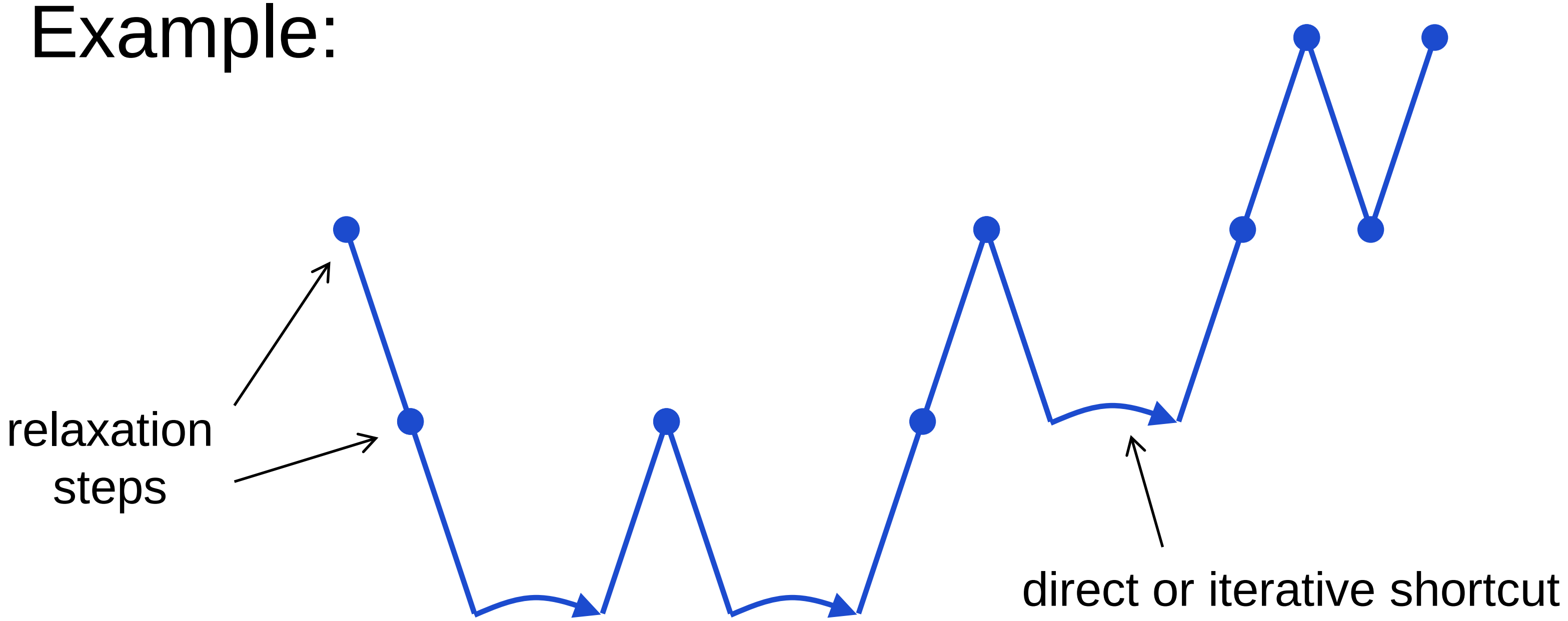


How many iterations?

Standard Approaches

Multigrid Cycles

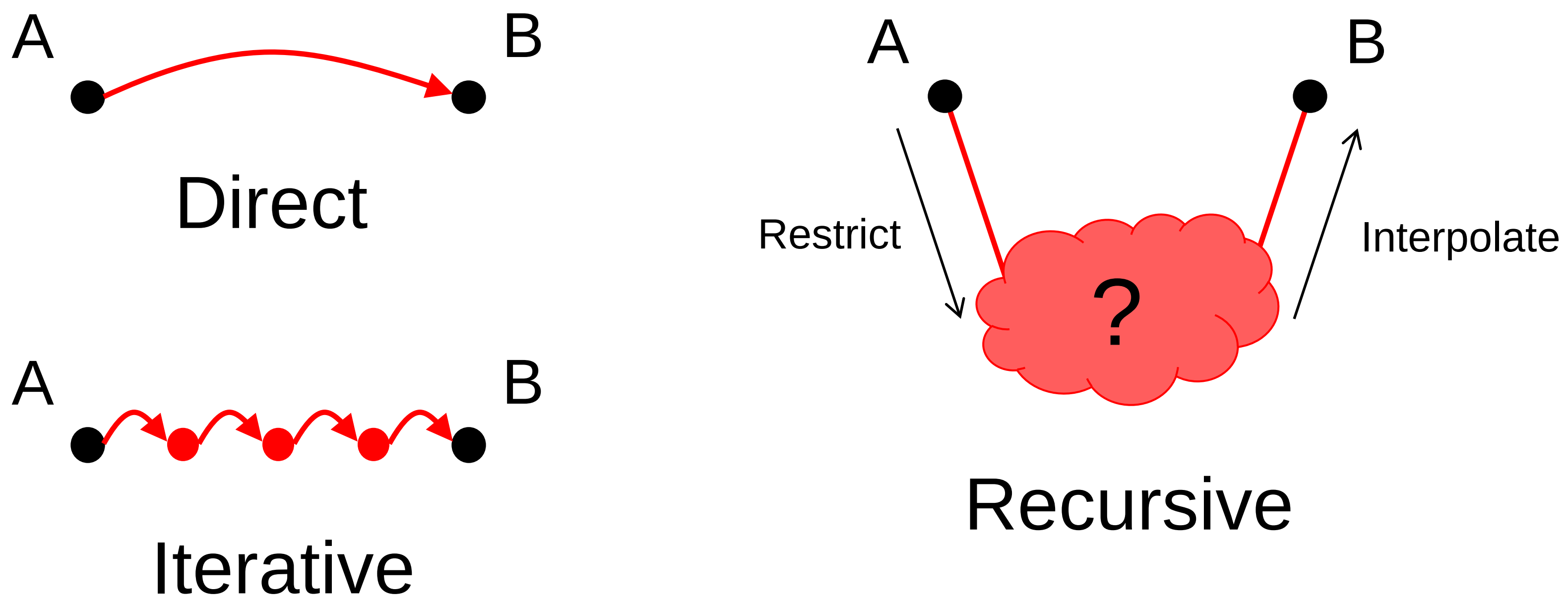
- Generalize the idea of what a multigrid cycle can look like
- Example:



- Goal: Auto-tune cycle shape for specific usage

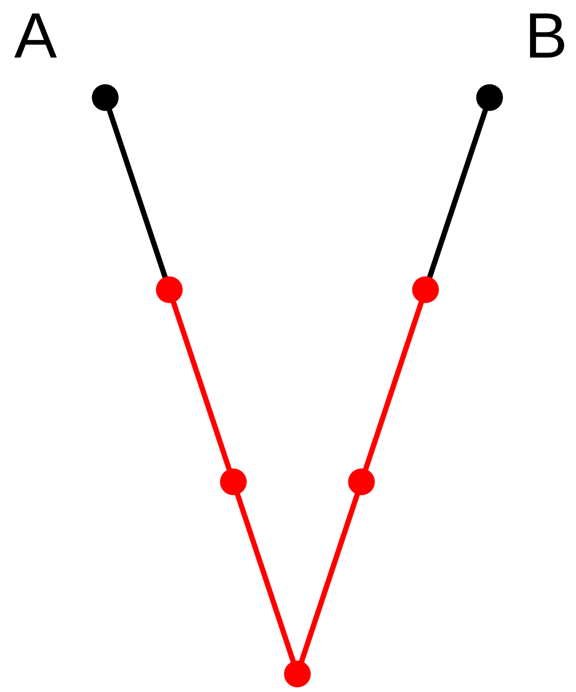
Algorithmic Choice in Multigrid

- Need framework to make fair comparisons
- Perspective of a specific grid resolution
- How to get from A to B?



Algorithmic Choice in Multigrid

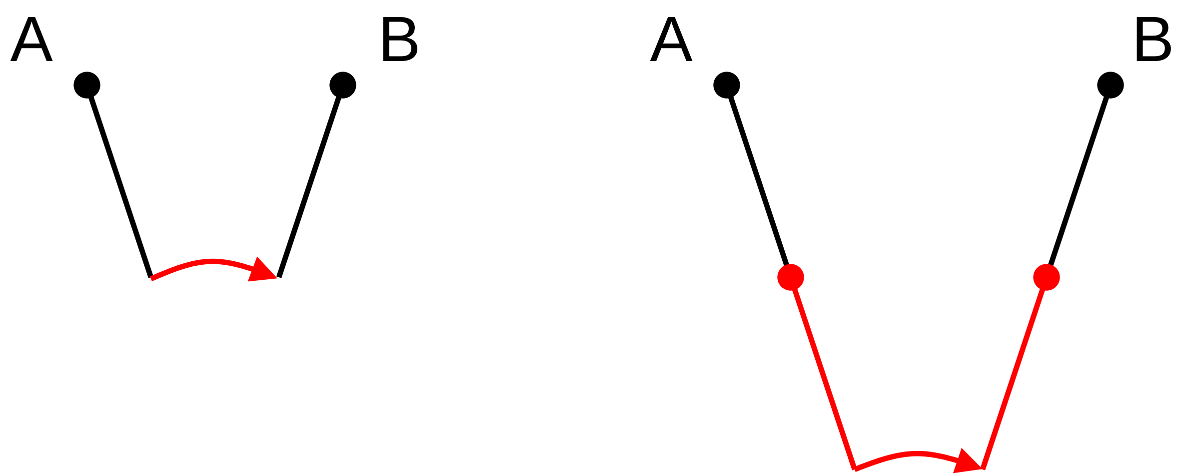
- Tuning cycle shape!
  - Examples of recursive options:



Standard V-cycle

Algorithmic Choice in Multigrid

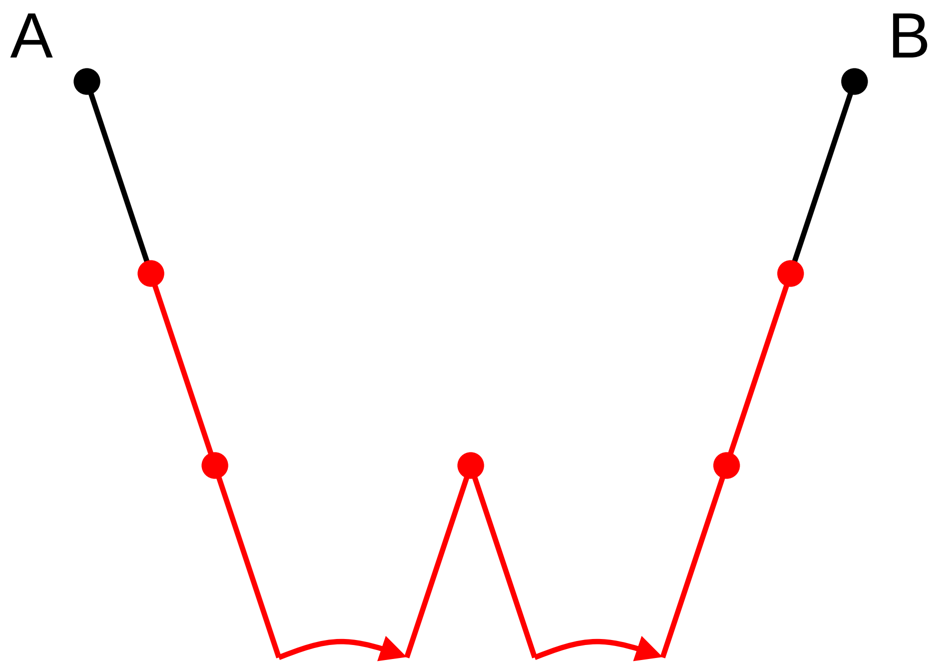
- Tuning cycle shape!
  - Examples of recursive options:



Take a shortcut at a coarser resolution

Algorithmic Choice in Multigrid

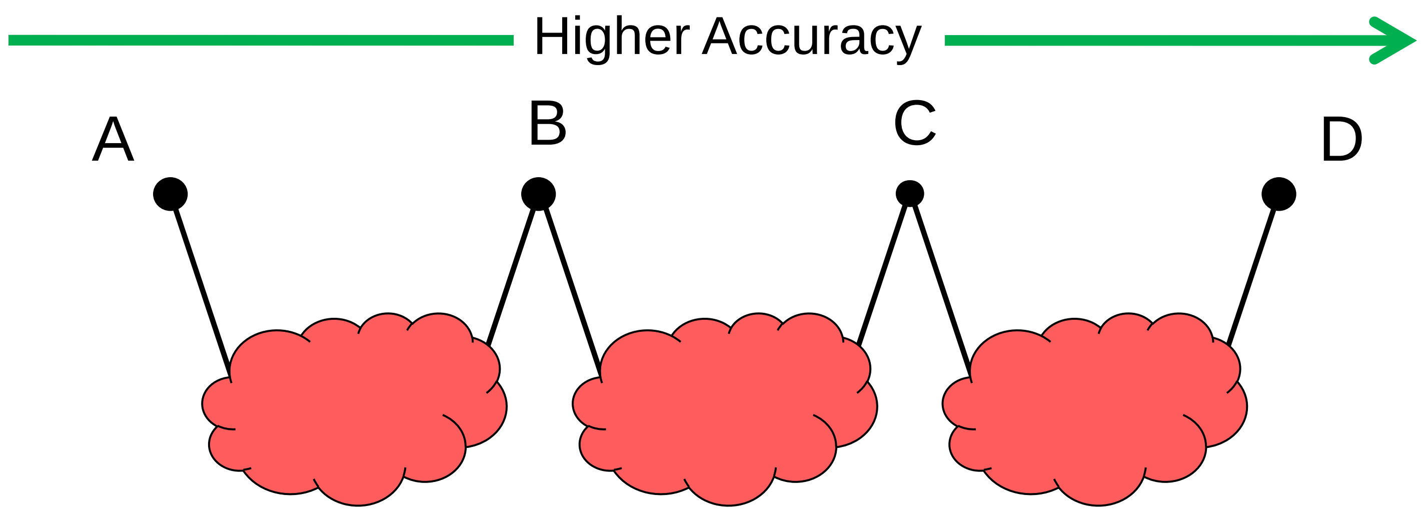
- Tuning cycle shape!
  - Examples of recursive options:



Iterating with shortcuts

Algorithmic Choice in Multigrid

- Tuning cycle shape!
  - Once we pick a recursive option, how many times do we iterate?



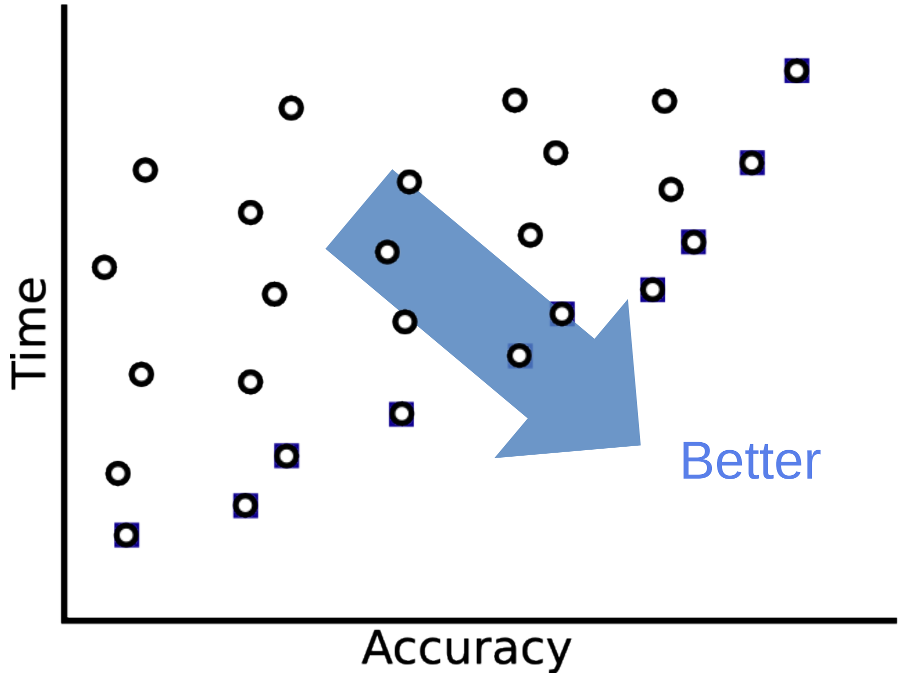
- Number of iterations depends on what **accuracy** we want at the current grid resolution!

Comparing Cycle Shapes

- Different convergence AND execution rates
- Need a way to make fair comparisons
- Measure accuracy: reduction of RMS error
  - Example: A cycle has accuracy level 10^3 if the RMS error of guess is reduced by a 10^3 factor
  - Must train on representative data
  - Imperfect metric: ignores error frequency
- Use accuracy AND time to make comparisons between cycle shapes

Optimal Subproblems

- Plot all cycle shapes for a given grid resolution:

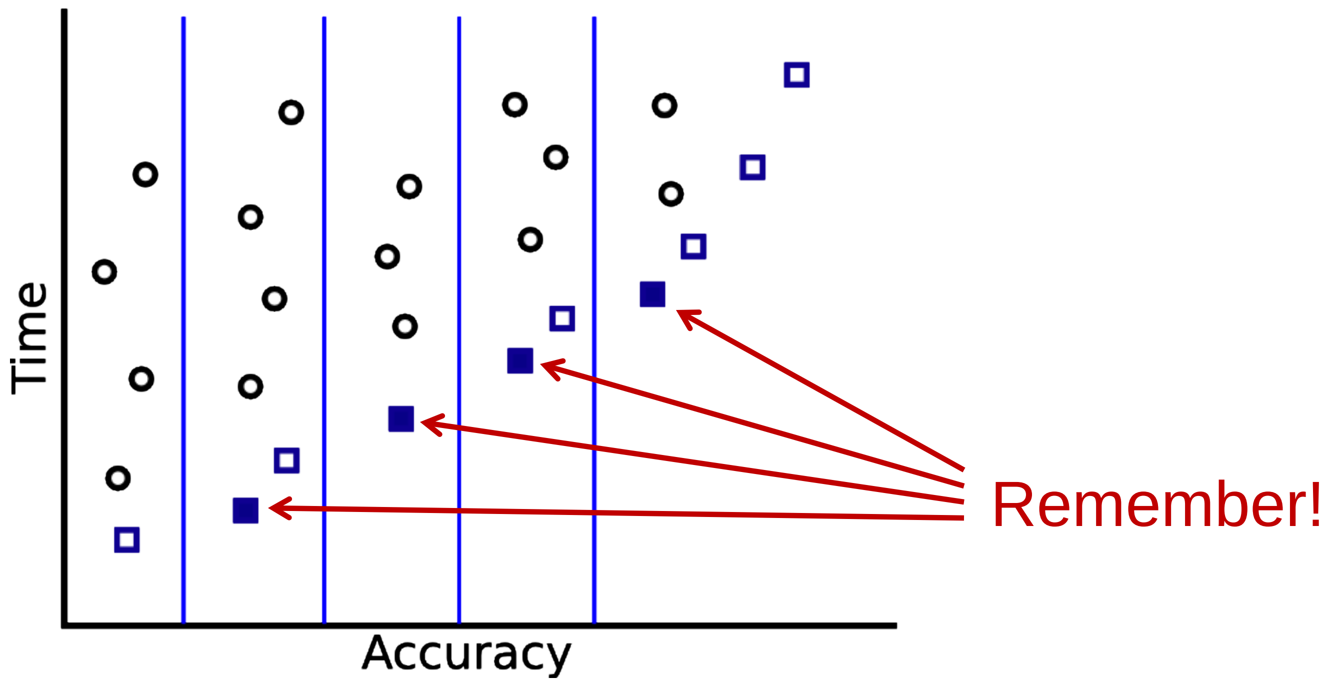


Keep only the optimal ones!

- Idea: Maintain a **family** of optimal algorithms for each grid resolution

The Discrete Solution

- Problem: Too many optimal cycle shapes to remember



- Solution: Remember the fastest algorithms for a discrete set of accuracies

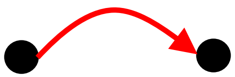
Use Dynamic Programming to Manage Auto-tuning Search

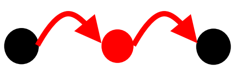
- Only search cycle shapes that utilize optimized sub-cycles in recursive calls
- Build optimized algorithms from the bottom up

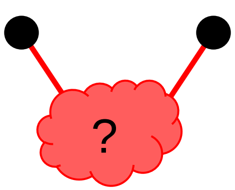
- Allow shortcuts to stop recursion early
- Allow multiple iterations of sub-cycles to explore time vs. accuracy space

Auto-tuning the V-cycle

```
transform Multigrid_k
from X[n,n], B[n,n]
to Y[n,n]
{
  // Base case
  // Direct solve

      OR

  // Base case
  // Iterative solve at current resolution

      OR

  // Recursive case
  // For some number of iterations
     // Relax
     // Compute residual and restrict
     // Call Multigrid_i for some i
     // Interpolate and correct
     // Relax
}
```







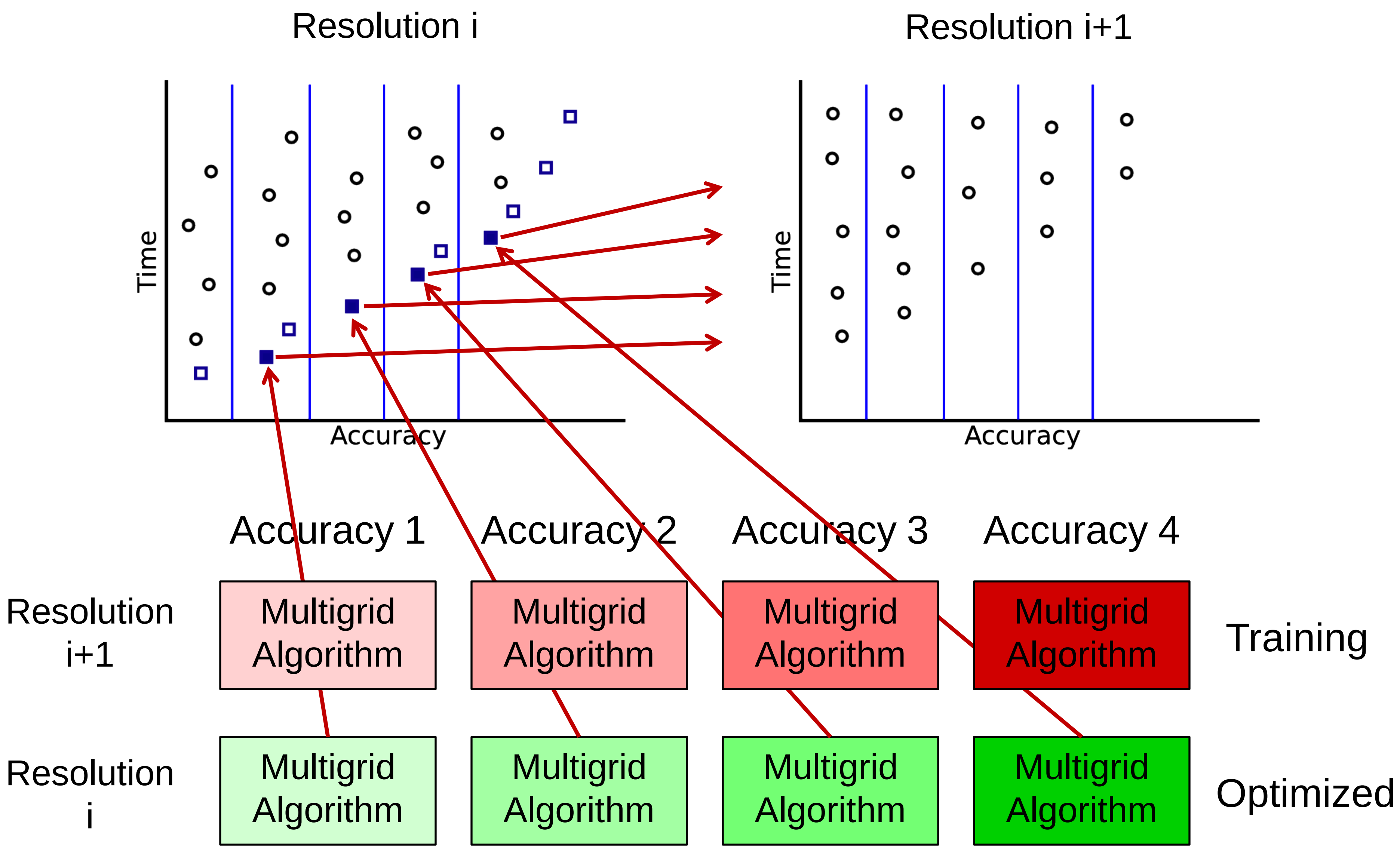
- Algorithmic choice
  - Shortcut base cases
  - Recursively call some optimized sub-cycle
- Iterations and recursive accuracy let us explore accuracy versus performance space
- Only remember “best” versions

Variable Accuracy Keywords

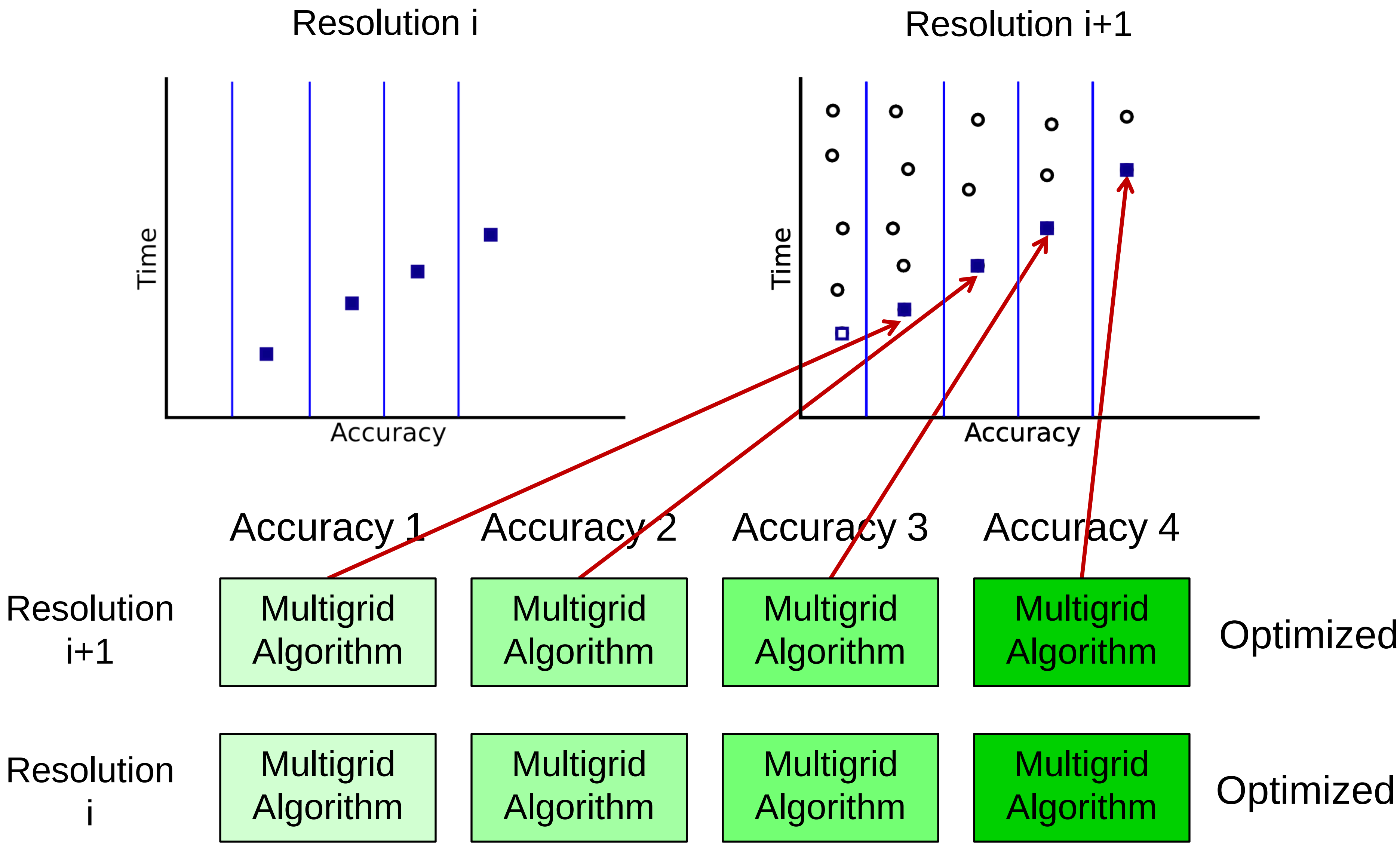
- **accuracy_variable** – tunable variable
- **accuracy_metric** – returns accuracy of output
- **accuracy_bins** – set of discrete accuracy bins
- **generator** – creates random inputs for accuracy measurement

transform Multigrid_k
from X[n,n], B[n,n]
to Y[n,n]
accuracy_variable numIterations
accuracy_metric Poisson2D_metric
accuracy_bins 1e1 1e3 1e5 1e7
generator Poisson2D_Generator
{
    …

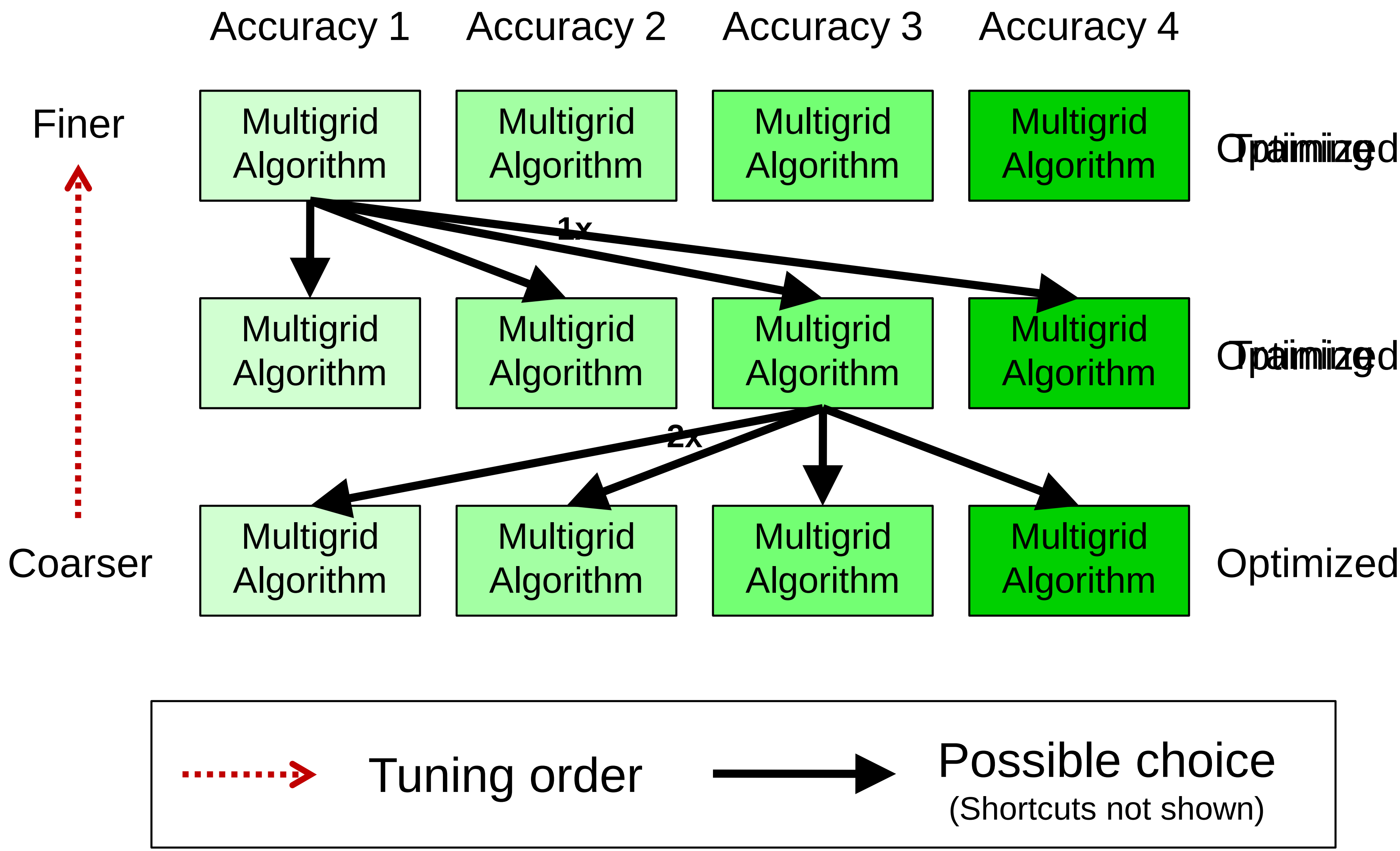
Training the Discrete Solution



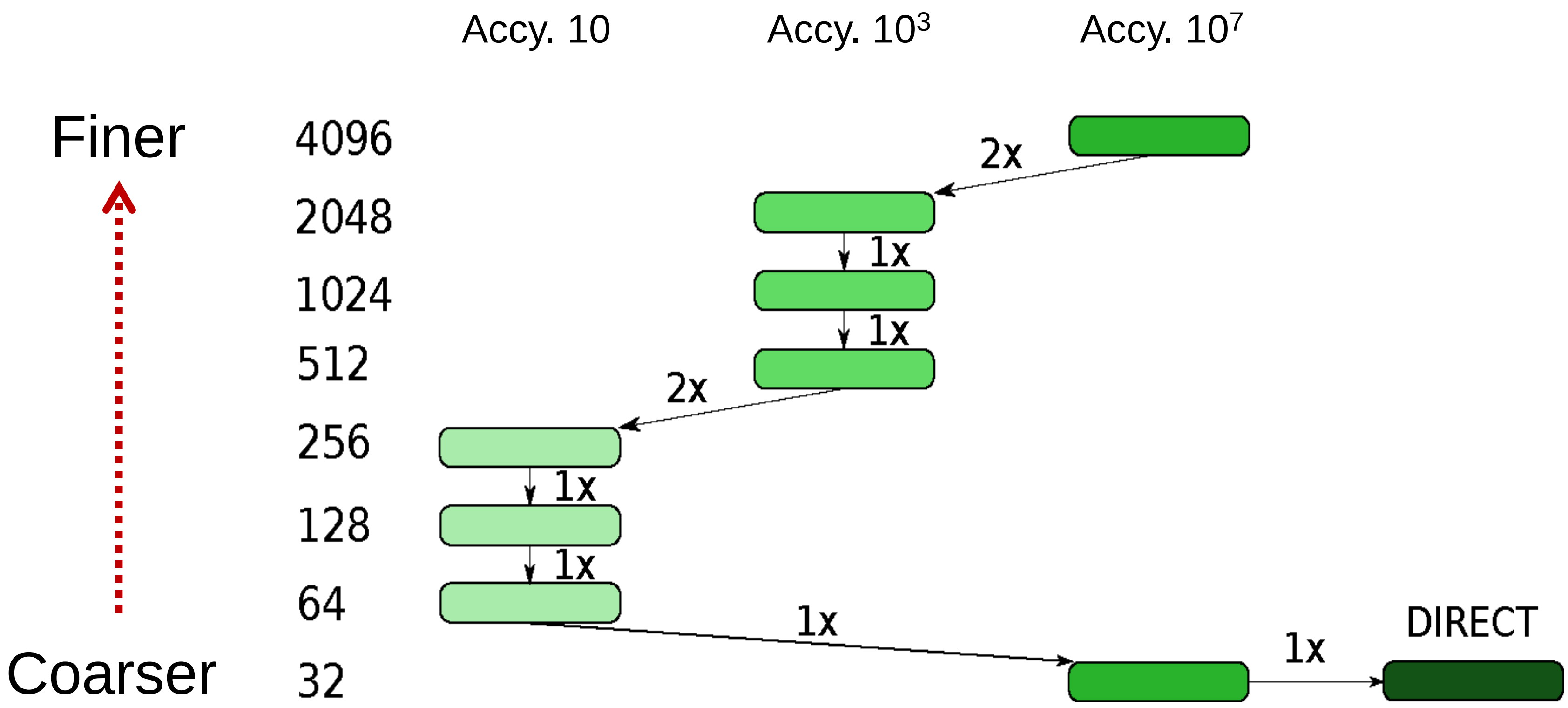
Training the Discrete Solution



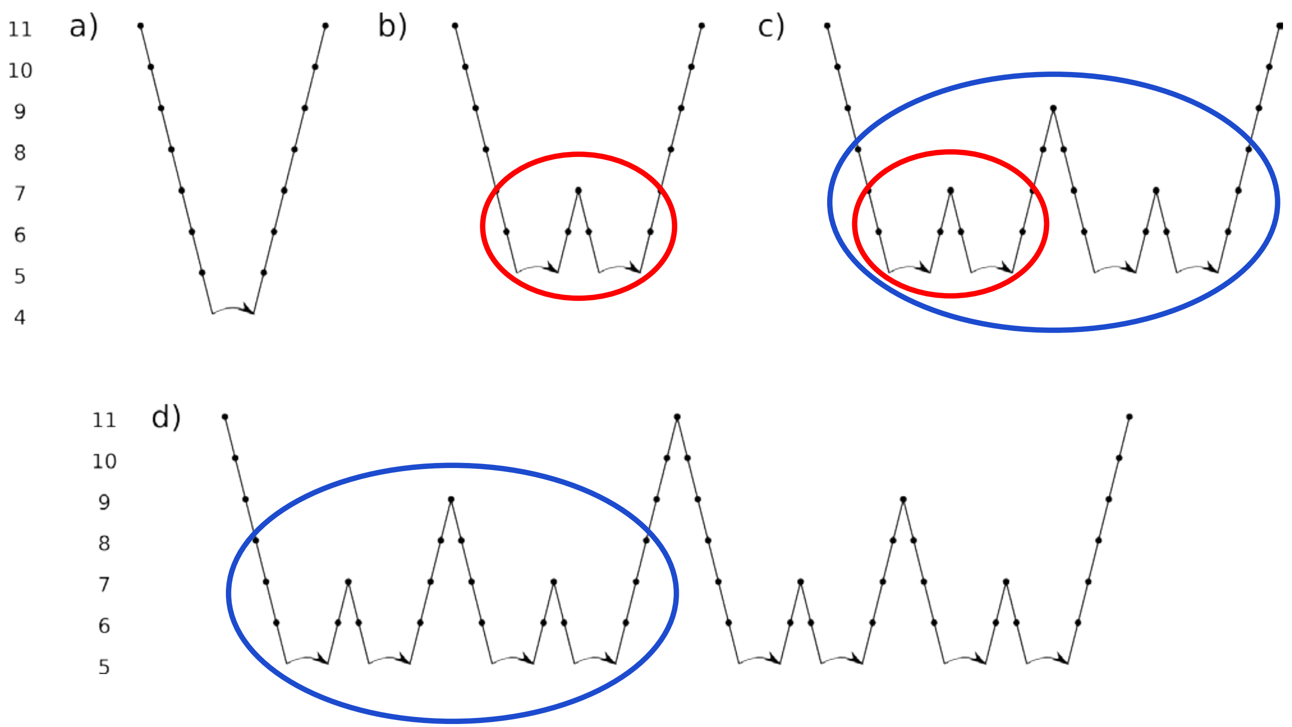
Training the Discrete Solution



Example: Auto-tuned 2D Poisson's Equation Solver



Auto-tuned Cycles for 2D Poisson Solver



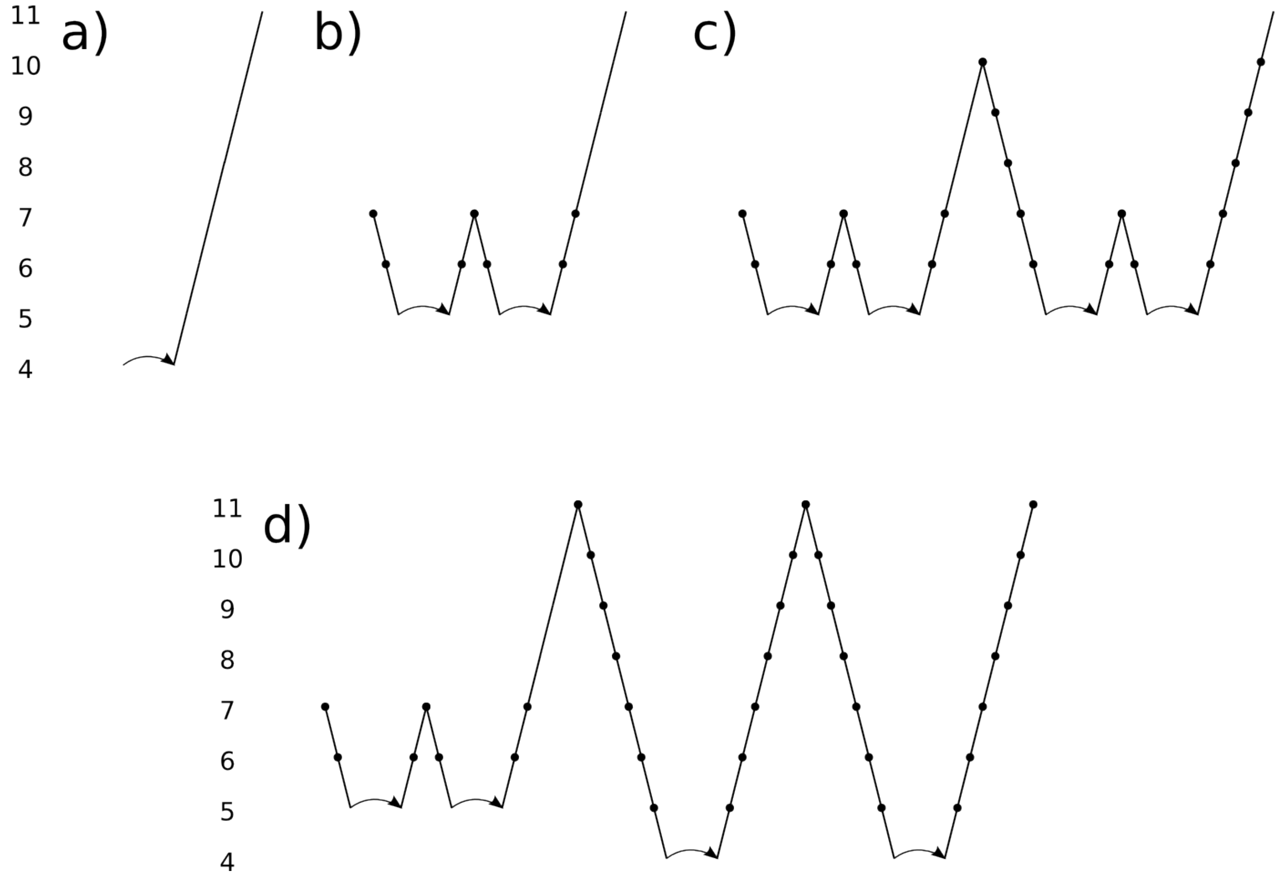
Cycle shapes for accuracy levels a) 10, b) 10^3, c) 10^5, d) 10^7

Optimized substructures visible in cycle shapes

Extension to Full Multigrid

- Build auto-tuned Full Multigrid cycles out of auto-tuned V-cycles
- Two phases:
  - Estimation phase: Restrict and recursively call **auto-tuned Full Multigrid** at **coarser grid resolution**
  - Solve phase: Interpolate and run **auto-tuned V-cycle** at **current grid resolution**
- Choose accuracy level of each phase independently
- Use dynamic programming

Auto-tuned Full Multigrid Cycles for 2D Poisson Solver



Cycle shapes for accuracy levels a) 10, b) 10^3, c) 10^5, d) 10^7

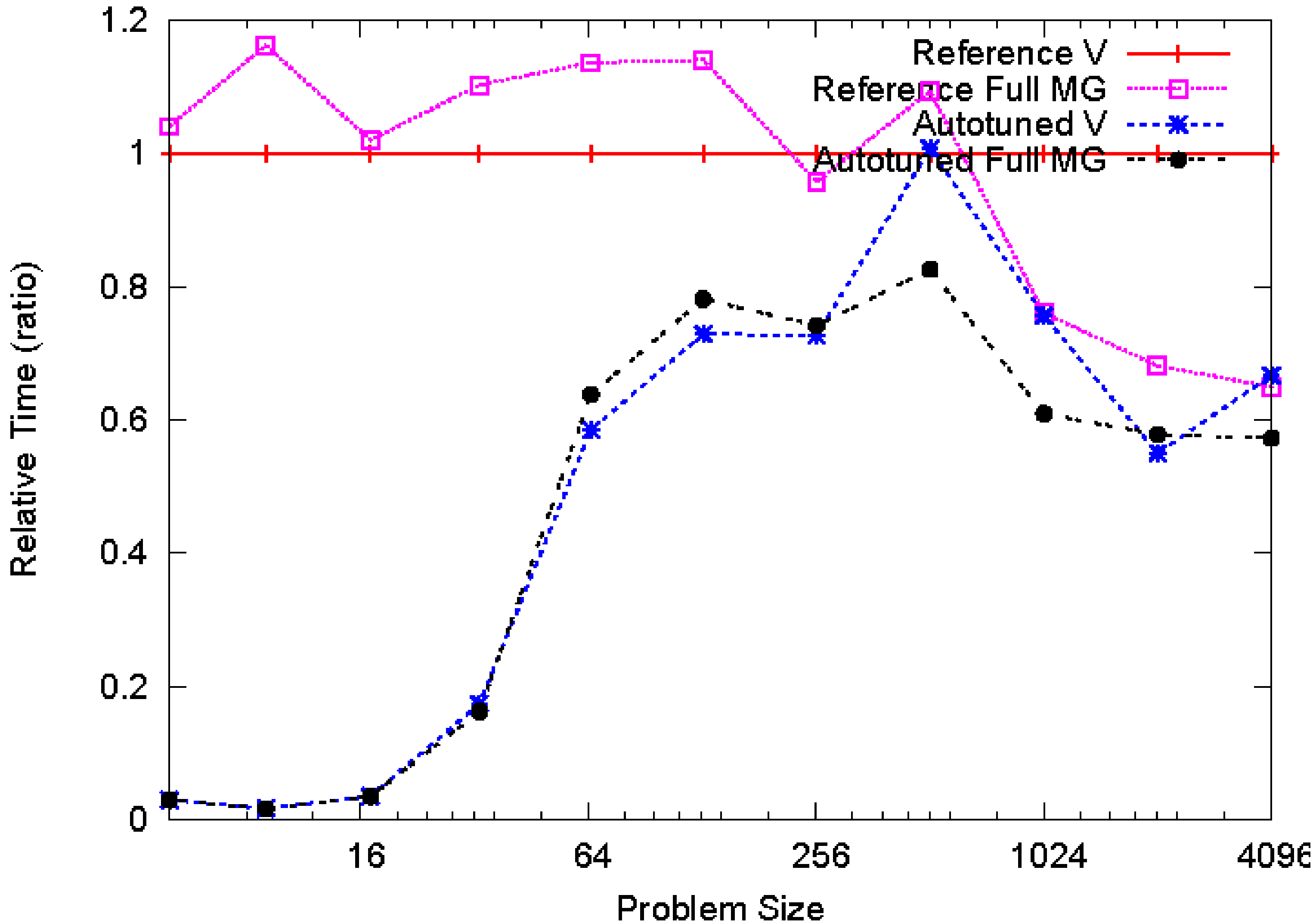
Benchmark Application: Solving 2D Poisson's Equation

- Solve 2D Poisson's Equation on random data (uniform over $[-2^{32}, 2^{32}]$) for problems of size 2^n for n = 2, 3, …, 12
- Reference Algorithms (also in PetaBricks):
  - Reference Multigrid – Iterate using V-cycle until accuracy target is reached
  - Reference Full Multigrid – Estimate using a standard Full Multigrid iteration, then iterate using V-cycle until accuracy target is reached

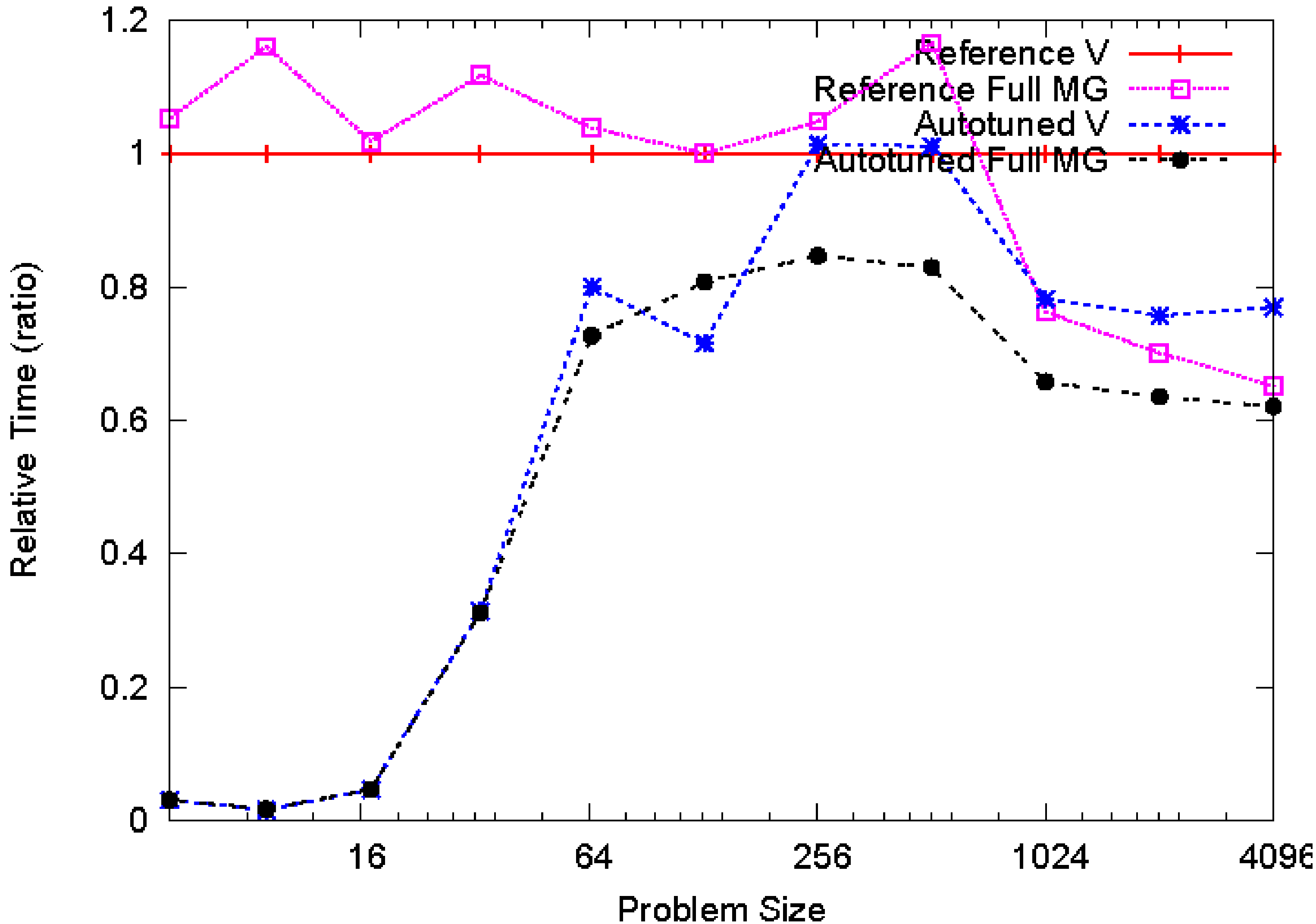
Performance Testbed

- Shared memory machines
  - Intel Harpertown – Two quad-core 3.2 GHz Xeons
  - AMD Barcelona – Two quad-core 2.4 GHz Opterons
  - Sun Niagara – One quad-core 1.2 GHz T1
- PetaBricks compiler still under development
  - Some low-level optimizations not yet supported (no explicit pre-fetching or SIMD vectorization)
  - Focus on tuning and comparing cycle shapes

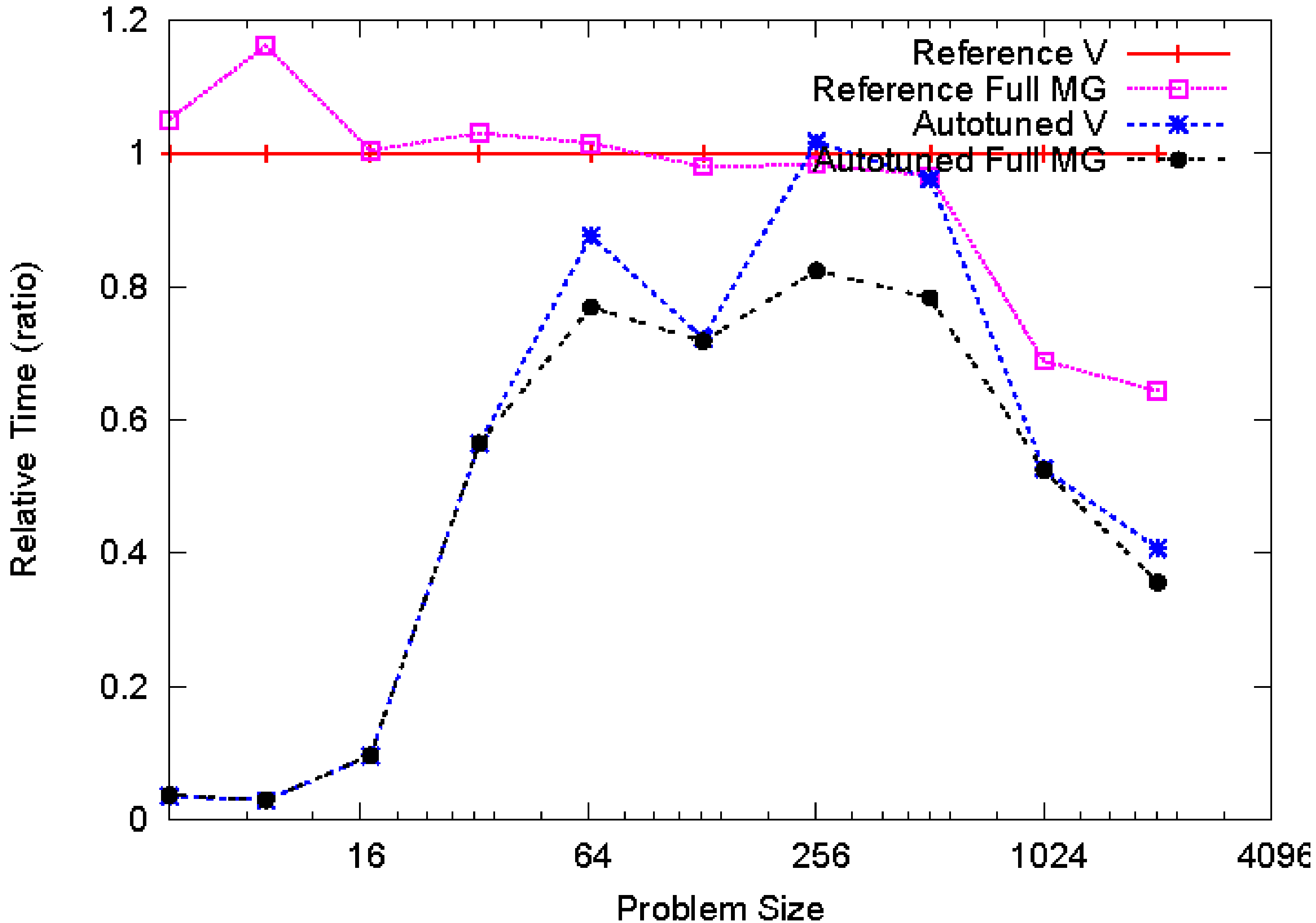
Impact of Auto-tuning
Intel Harpertown (2 Sockets, 8 Cores)



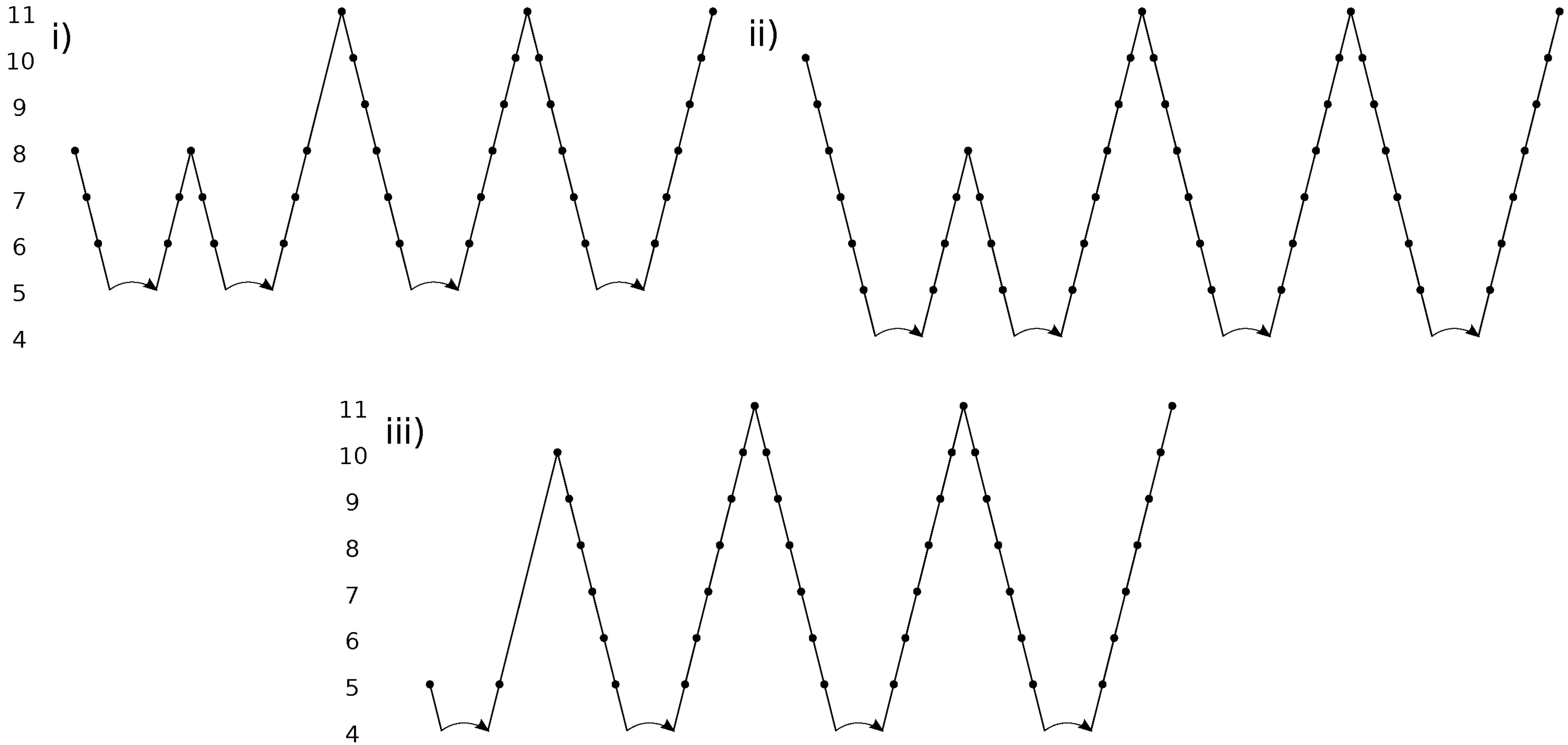
Impact of Auto-tuning
AMD Barcelona (2 Sockets, 8 Cores)



Impact of Auto-tuning
Sun Niagara (4 Cores, 32 Threads)



Tuned Cycles Across Architectures



Tuned cycles to achieve accuracy 10^5 at resolution 2^{11}
i) Intel Harpertown ii) AMD Barcelona iii) Sun Niagara

Selected Related Work

- Auto-tuning Software:
  - FFTW – Fast Fourier Transform
  - ATLAS, FLAME – Linear Algebra
  - SPARSITY, OSKI – Sparse Matrices
  - STAPL – Template Framework Library
  - SPL – Digital Signal Processing
- Tuning Multigrid:
  - SuperSolvers – Composite Linear Solver
  - Sellappa and Chatterjee (2004), Rivera and Tseng (2000) – Cache-Aware Multigrid
  - Thekale, Gradl, Klamroth, Rude (2009) – Optimizing Interations of V-Cycles in Full Multigrid

Future Work

- Add support for auto-tuning other aspects of multigrid
  - Tuning of relaxation, interpolation, and restriction operators
  - Low-level optimizations: explicit prefetch and vectorization
- Add support for tuning data movement in AMR
  - Parameterize tuned subproblems by data location in addition to size and accuracy
  - Try different data layouts during recursion

General PetaBricks Future Work

- Dynamic choices during execution
- Support for other parallel architectures
  - Distributed memory machines
  - Heterogeneous clusters (e.g. CPU + GPGPU)
- Sparse Matrix support
  - Auto-tune sparse matrix storage format
    - e.g. CSR, CSC, COO, ELLPACK
    - register block sizes, cache block sizes

Conclusion

- Auto-tuning with PetaBricks
  - Algorithmic choice
  - Variable accuracy
- Auto-tuning Multigrid Cycles
  - Construct more efficient multigrid solvers
  - Use dynamic programming
  - Speedup shown over reference algorithms

Thanks!

http://projects.csail.mit.edu/petabricks/