# Toward a solution to the high Weissenberg number problem

**David Trebotich**[*1]

[1] Center for Applied Scientific Computing, Lawrence Livermore National Laboratory, P.O. Box 808, L-560, Livermore, CA 94551, USA.

We present results of a numerical study of high Weissenberg number flows using the method of Trebotich, Colella and Miller, J. Comput. Phys. vol. 205 (2005). We consider benchmark viscoelastic flow of an Oldroyd-B fluid in four to one sudden contractions for $We = 1, 10$, $Re = 1$. We conclude that (i) underlying wave behavior in the solution for flows at critical and supercritical elastic Mach number prevents the solution from reaching a steady-state; and (ii) high resolution is needed to resolve structure in the solution downstream from the re-entrant corner in order to determine convergence.



Historically, viscoelastic fluids have suffered from the inability of numerical methods to compute solutions beyond a frustratingly low value of elasticity in the fluid. This is known as the "High Weissenberg Number Problem" (HWNP), a benchmark problem for viscoelastic flow in a sudden contraction channel flow [1, 2, 3, 4, 5]. Computational symptoms of the problem include large normal stresses near geometric singularities and worsening of the problem with grid refinement. The critical dimensionless parameter is the Weissenberg number, We, which is the ratio of the polymer relaxation time to the advective time scale. Another dimensionless parameter which defines the flow, and perhaps more critical to solving the HWNP, is the elastic Mach number, Ma. The critical Mach number identifies the parameters at which elastic shear wave propagation is admitted in the fluid and the mathematical system switches from hyperbolic to elliptic, that is, the eigenvalues of the system become imaginary [6, 7, 8]. This continues to be an outstanding research issue as we have just begun to scratch the surface of understanding the interplay between viscous and elastic effects.

We present two results that give insight into the solution to the HWNP. First, we have shown that the limiting behavior of a Maxwell fluid (zero solvent viscosity) reveals purely elastic shear wave propagation in an incompressible fluid [7]. As viscosity is added to the system this wave behavior becomes dampened to a point where the HWNP parameter space is reached. Though it looks like a steady-state problem there remains underlying wave behavior. See Figure 1. Second, we have shown that previous simulations of benchmark viscoelastic flows have failed to reach the asymptotic regime for solution convergence. Our grid refinement studies demonstrate a need for very high resolution near the geometric singularity of the flow, revealing localized structure in the solution that may or may not be a numerical artifact [9]. See Figures 2a and 2b.

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* Corresponding author: e-mail: trebotich1@llnl.gov, Phone: +01 925 423 2378, Fax: +01 925 422 6287

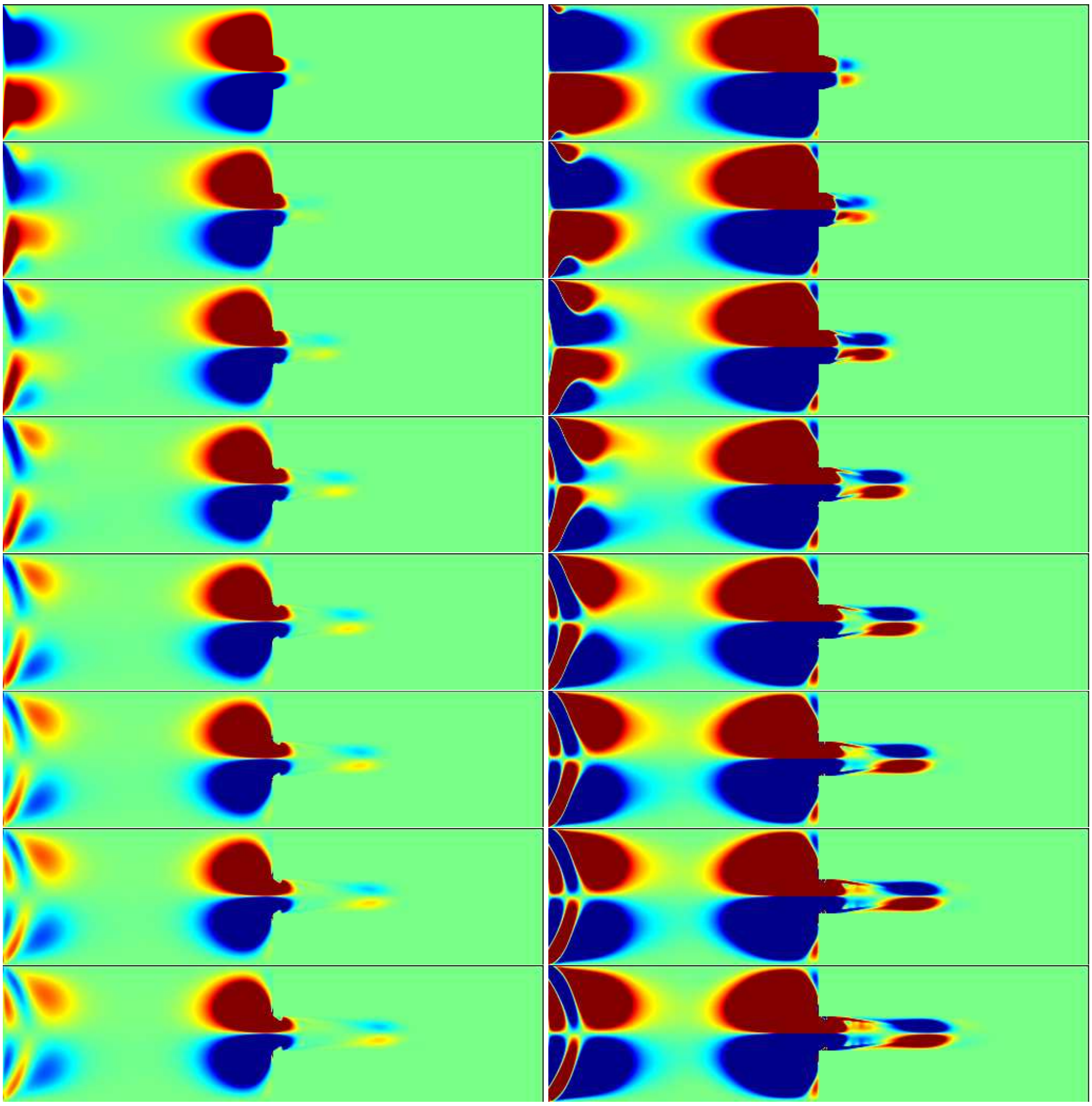

**Fig. 1** Demonstration of underlying wave behavior in transverse velocity for benchmark flow in sudden contraction channel. (L) Data has been scaled by .1 to see underlying effect; (R) Data has been scaled by .01 to see underlying effect.

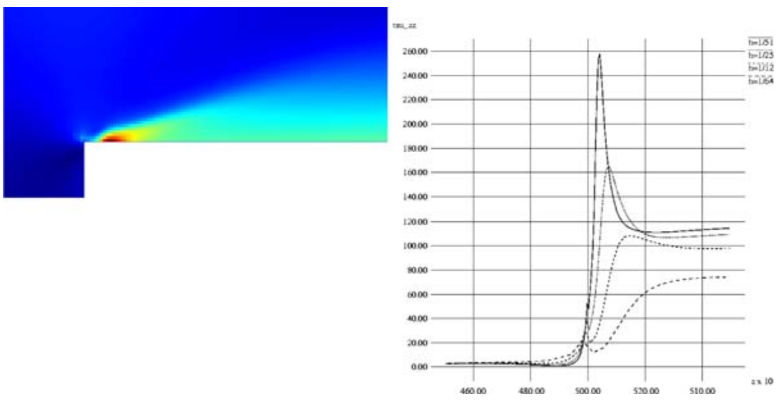

**Fig. 2** (L) Close-up view of of unresolved structure in $\tau_{zz}$ a short distance downstream of re-entrant corner for $We=1$, $Re=1$ benchmark, -13.14 (blue) to 257.55 (red), $h=1/512$. (R) $\tau_{zz}$ along contraction wall for $We=1$, $Re=1$ benchmark. Corner is located at $z=0.5$. More grid refinement is needed to resolve the feature.